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Account of a MS. Table of Twelve Figure Logarithms of Numbers from 1 to 120,000, *calculated by the late Mr. John Thomson, of Greenock, and recently presented to the Royal Astronomical Society by his sister, Miss Catherine Thomson.* Drawn up, at the request of the Council, by J. W. L. Glaisher, M.A.

§ 1. *Description of the MSS.; Mode of Calculation, &c.*

In the course of last June (1873) Mr. Andrew Williamson, of Greenock, wrote to the Secretaries of the Royal Astronomical Society, with reference to a large MS. Table of Logarithms, calculated by the late Mr. John Thomson, of that town, and which, under certain circumstances, his sister, Miss Catherine Thomson, was willing to present to the Society. Mr. Williamson's letter was accompanied by one from Mr. Thomas Cranston, teacher of mathematics and navigation at Greenock, giving a short account of the contents of the MSS., from which it appeared that they consisted of a twelve figure Table of Logarithms, of numbers from 1 to 120,000, apparently the result of an independent calculation, together with a considerable portion of the work. Having been requested by the Council to act on their behalf in the matter, I wrote to Mr. Williamson and expressed an opinion that there was but slight chance of anyone being likely to undertake the publication of the table, which would necessarily entail a heavy pecuniary outlay, with very small return, but that the MSS. might be of high interest for purposes of reference. Thereupon, Miss Thomson at once presented the whole of the MSS. to the Society, expressing, through Mr. Williamson, her belief that she was thus rendering her brother's work as useful and accessible as possible, and was adopting the course which, had he lived, would have been most in accordance with his wishes. For this generous gift the thanks of the Society were returned to Miss Thomson, and also to Mr. Williamson and Mr. Cranston for their good offices, at the ensuing meeting, in November 1873.

Before describing the MSS. at length, which on examination appear to be a very valuable addition to the Society's library, it is proper to state the following facts relative to Mr. Thomson's life with which, at my request, I have been kindly furnished:—Mr. John Thomson was born at Strachur, Argyleshire, in 1782. His father, Dougal Thomson, farmer, married Margaret McKinlay, by whom he had eleven children, John being the second. He was educated at Strachur till he was twelve years of age, when he went to Greenock, and was placed under the care of the late Colin Lamont, teacher of mathematics and navigation, from whom he imbibed a taste for mathematical studies. For many years he occupied the position of clerk to a firm in Greenock, but subsequently he commenced the business of an accountant. He died on December 31, 1855, and was buried in Duncan Street burying-ground, Greenock, where his grave is marked by no stone or monument. He was never married, and for some time before his death suffered from failing health. These few sentences contain all the facts which Mr. Williamson and Mr. Cranston could collect relative to Mr. Thomson's life. His sister, the donor of the tables, being his junior by more than twenty years, knows nothing of his early studies, and very little of his pursuits in the prime of life. The work undertaken by Mr. Thomson, and the manner in which he has performed it, testify more clearly than could anything else to his steady and methodical habits.

The MSS. consist of four volumes of results, eight volumes containing portions of the calculation, and three small covers and a tray containing slips. A more exact enumeration, giving also the letters with which I have marked (in red ink) the different parts of the MSS., for purposes of reference, is as follows:—

A. (4 Vols.) All the results; viz., a complete table of logarithms of numbers from unity to 120,005 (besides the logarithms of 744 higher numbers at irregular intervals, the highest being log 123,187) to 12 places of decimals. Vol. 1 (marked A_1, size of page 9 in. by 3·5 in.) extends from 1 to 25,694; and the contents of the other three vols. (marked A_2, A_3, A_4, quarto size) are 22,695 to 60,179; 60,180 to 91,200; and 91,201 to 120,005, with the 744 additional logarithms just mentioned.

a. (2 Vols.) Logarithmic differences. Vol. 1 (marked a_1, small quarto) contains the last nine figures of the logarithms, with first and second differences, for numbers from 29,561 (erroneously written on the cover 29,651) to 57,401; and vol. 2 (marked a_2, quarto) contains the last nine or six figures of the logarithms, with first and second differences, for numbers from 57,401 to 120,001 (the last nine figures are given as far as 71,201).

b. (2 Vols.) Logarithms of multiples. Vol. 1 (marked b_1, octavo) is entitled "Preparative Tables of Logarithms,

1st Part"; and vol. 2 (marked b_2, octavo) "Preparative Tables of Logarithms, 2nd Part, continuations." They contain logarithms of multiples of 3, 5, 7 . . . and of prime numbers up to 55,691 (of the higher numbers the logarithms of the doubles only are given).

c. (3 Vols.) Logarithms of higher multiples required in the portion of the table from 40,000 to 120,000. Vol. 1 (marked c_1, small quarto) is entitled "Logarithms, multiples of 3, Nos. 40,000 to 120,000," and contains logarithms of multiples of 3, the lowest multiple being $13{,}334 \times 3 = 40{,}002$, and the highest $40{,}001 \times 3 = 120{,}003$; Vols. 2 and 3 (marked c_2 and c_3, small quarto) are respectively entitled "Logarithms, multiples of 5 to 101, Nos. 40,000 to 120,000," and "Logarithmic Multiples of 103 to 15,013, Nos. 40,000 to 120,000," and are similar in their contents to c_1, the primes whose logarithmic multiples are given being indicated in the titles.

d. Three small covers of stiff paper, and a tray, containing slips with the logarithms used in the formation of *b* and *c* written upon them. The three covers (marked d_1, d_2, d_3, size 8 in. by $2\frac{1}{2}$ in.) are entitled respectively, "Logarithmic Slips, Nos. 1 to 2,000," "Logarithmic Slips, Nos. 2,000 to 4,000," "Logarithmic Slips, Prime Nos. above 4,000." The little cardboard tray (marked d_4, same size as the covers) contains the logarithms of the lower numbers, written each on a separate slip of cardboard.

e. (1 Vol.) A small MS. of 34 leaves, octavo, entitled, "Numbers distinguishing primes from composites," and giving the factors (2 and its powers being omitted) of the numbers up to 21,460.

There are besides several other loose MSS. that will be referred to further on (see the end of this section); one, consisting of 16 small leaves stitched together, gives several powers of a few primes and their logarithms.

The general method followed by Mr. Thomson in his calculations is evident from the above enumeration, viz., the logarithms of the primes having been obtained, the logarithms of the composite numbers were deduced by addition, and the whole was verified by differences. There is not, however, among the MSS. anything that relates to the calculation of the logarithms of the primes, nor is there to be found a single formula or piece of algebra having any reference whatever to logarithms. All the papers that the Society possesses merely concern the formation of the logarithms of the composite numbers by the addition of the logarithms of their factors, and there is no direct evidence of the method pursued with regard to the primes. For reasons that will be stated hereafter, however, there can be but little doubt that the logarithms of these were also calculated *de novo*, although the work has been lost. Leaving, then, for the present, the primes out of the question, and assuming the author to have been in

the possession of their logarithms, the rest of the procedure is clear. The first thing was to transcribe the logarithms of 2, 3, 5 . . . . upon separate slips of paper, and so by addition obtain the logarithms of the composite numbers up to some limit, say 1,000; then, having obtained the logarithms of these first thousand numbers, by adding them to log 3, we obtain the logarithms of the first thousand multiples of 3, by adding them to log 5, of the first thousand multiples of 5, and so on. The slips so used are contained in *d*,* and the additions performed by means of them are given in the 5 vols. *b* and *c*. It would be impossible to explain the exact course pursued, which seems to have been guided by no fixed rule. Thus, in b_1 we have logarithms of multiples of 3 up to 385×3 (= 1,155), of 5 up to 207×5 (= 1,035), of 7 up to 358×7 (= 2,506), of 11 up to 156×11 (= 1,716), of 13 up to 151×13 (= 1,963)..., of (say) 11,939 up to $7 \times 11{,}939$ (= 83,573), &c., &c. Many multiples are, however, omitted—thus, in 3, 260×3; 262×3; 263×3; 267×3; 268×3 are consecutive multiples; in 7, 31×7; 32×7; 34×7; 37×7; 38×7; 40×7 are consecutive, and so on. No particular principle seems to have governed these omissions, the object of which, of course, was to prevent the same logarithm being calculated several times over, as a multiple of each of its factors; but no attempt was made to carry this out rigorously, as many logarithms do, in fact, appear twice over, and some even more frequently. In b_2 we have the continuation of the multiples in b_1; thus, the multiples of 3 begin at 386×3 (= 1,158), and end at $13{,}376 \times 3$ (= 40,128); the multiples of 5 begin at 209×5 (= 1,045), and end at $8{,}001 \times 5$ (= 40,005), and so on. The vols. c_1, c_2, c_3, contain the higher multiples required in the portion of the table from 40,000 to 120,000. The vols. b_1 and b_2 do not contain multiples that exceed 40,000 of

* A more exact enumeration of the contents of the covers, &c., marked *d*, is as follows: d_1 contains 45 slips, giving the logarithms of the numbers from 1 to 2,000 (15 places); nearly all have writing on both sides, and when this is the case each gives about 50 logarithms; a few of the slips near the beginning having become worn out by use, are re-written, and so appear twice. d_2 contains 40 similar slips, extending from 2,000 to 4,000. d_3 contains 51 slips, giving the logarithms of primes, the lowest being 4,001, and the highest 30,319 (12 places). Each slip has been folded into creases after every logarithm, as would be required in adding them consecutively to the constant logarithms at the tops of the pages in *b*; and even now the slips present a wavy appearance, which, not less than their worn aspect, shows how much they have been used. The little tray d_4 contains 60 small slips of cardboard, of which 25 have a number and its logarithm on one side, and the other blank; and 35 have a pair of numbers and their logarithms, one on each side (in these cases the second number which appears is the number whose logarithm is to be found on the back, thus re-written to avoid the necessity of turning over the slip to see what is given there). The numbers range from 2 to 341, running regularly (multiples of 10 omitted) to 65; thence by uneven numbers to 95, above which there are only fourteen numbers (12 or 15 places), and a few occur twice. These logarithms of low numbers were doubtless thus written separately on cards on account of their more frequent use.

numbers up to 11,489, but above this limit the multiples are given as far as 80,000, and for numbers above (about) 20,000 they are given to over 100,000 (in fact, the first 8, 7, 6, 5... multiples are given, with omissions, according to the magnitude of the numbers); and c_1, c_2, and c_3 are therefore complementary to them. The factor table e was no doubt employed to determine what multiples might be omitted, besides being otherwise useful. It is no fault whatever that the multiples chosen are not subject to exact rules; it would clearly be a great deal more difficult and inexpedient to devise and carry out a scheme by which no composite number should appear twice, than to proceed as Mr. Thomson has done, leaving out only what he saw would be calculated in another part of the work, though of course making these omissions capriciously.

The logarithms being thus obtained, they were entered in order in the books A, and the last nine and six figures of them were also entered in a_1 and a_2, and the first and second differences taken, the errors so found being also corrected in A. An examination of the errors thus corrected renders it probable that the entries were made in A and a at the same time, but there is no reason why in certain parts of the table the logarithms should not have been copied, a good many at a time, from A into a. That such was not invariably the case is shown by the fact that certain logarithms (generally those of primes) have clearly sometimes been entered, both in A and a, at a different time from the majority of logarithms in their neighbourhood. At all events, all the logarithms above that of 29,561 have been second-differenced, and the errors so found carefully corrected in A; also, as it is not likely that Mr. Thomson should have commenced to difference at the number 29,561, it is fair to conclude that the earlier logarithms (from perhaps that of 10,000) were also second-differenced in a volume of the MSS. that has been lost.

It now remains to consider whence Mr. Thomson derived the logarithms of the primes. In the first place, there can be little doubt that the logarithms of the primes under 4,000 (which in b_1 are given to 15 places) were the result of an original calculation, as, I believe, there is no printed table that contains such high primes to so many as 15 decimal places; and the internal evidence also strongly supports this opinion. In 1717, Abraham Sharp published his "Geometry Improved," in which he gave the logarithms of all primes less than 1,100 to 61 places, and this table has several times been reprinted in other works, generally to a less number of figures, and with the addition of the logarithms of the intervening composites. Thus Callet gives the logarithms of the first 1,200 numbers to 20 places, and I accordingly read Mr. Thomson's logarithms of primes up to this extent with a Callet of 1795, the 20 places being supposed contracted to 15. There are 196 primes below 1,200, and (assuming Callet in all cases to be accurate) the discrepancies were as follow:—

The MS. was correct			in 8	instances
„	„	incorrect by 1	„ 21	„
„	„	„ 2	„ 20	„
„	„	„ 3	„ 31	„
„	„	„ 4	„ 42	„
„	„	„ 5	„ 35	„
„	„	„ 6	„ 22	„
„	„	„ 7	„ 14	„
„	„	„ 8	„ 2	„
„	„	„ 9	„ 1	„

The errors are of course expressed in units of the last (fifteenth) place. To determine whether the errors increased in amount as the numbers became larger, the 196 primes were divided into two groups, the one containing the first 98 (viz., from 2 to 521 inclusive) and the other, the second 98 (viz., from 523 to 1193), with the following results :—

In the first group of 98

The MS. was correct			in 7	instances
„	„	incorrect by 1	„ 17	„
„	„	„ 2	„ 13	„
„	„	„ 3	„ 15	„
„	„	„ 4	„ 15	„
„	„	„ 5	„ 18	„
„	„	„ 6	„ 7	„
„	„	„ 7	„ 6	„

In the second group of 98

The MS. was correct			in 1	instance
„	„	incorrect by 1	„ 4	instances
„	„	„ 2	„ 7	„
„	„	„ 3	„ 16	„
„	„	„ 4	„ 27	„
„	„	„ 5	„ 17	„
„	„	„ 6	„ 15	„
„	„	„ 7	„ 8	„
„	„	„ 8	„ 2	„
„	„	„ 9	„ 1	„

From these lists the increase in inaccuracy may be fairly estimated. Mr. Thomson's values were invariably too small, with only three exceptions, viz., log 79 and log 487 were too large by unity, and log 223 was too large by 5.

This comparison points very clearly to Mr. Thomson's values being the result of an original calculation, and it also shows how little he has availed himself of existing works as a means of verification. Briggs (1624) gives the logarithms of the first 20,000 numbers to 14 places, and the logarithms of the first 1,200 numbers have been published to a great many places in several works, but no fifteen figure table has appeared of numbers between 1,200 and 4,000. Also, it is unlikely that results so inaccurate in the last figure should have been derived from a printed source. As the basis of a twelve figure table, they are just such as we should expect them to be, and are abundantly accurate for the

purpose, but it is not probable that anyone would print them without reducing the number of places. Had Mr. Thomson known of Sharp's table (which he would have had no difficulty in consulting, as it is reprinted in Sherwin, Hutton, and Callet), it seems almost inconceivable that he should not have corrected his own by means of it: no one can imagine a computer wilfully using erroneous values merely because they are the results of his own calculation, when he has the correct ones before him—so that there is strong evidence that the logarithms of the primes under 4,000 were calculated *de novo*, without help from, or reference to, any published table. This is confirmed by the rest of the MSS., in which I can find no trace whatever of any assistance having been derived by Mr. Thomson from any published work or source independent of his own calculation. It seems strange that anyone should have undertaken to do with his own hand so large a piece of work without having previously studied the existing tables; but I am inclined to believe, that, having read with interest the properties of logarithms, and met with an account of the mode of construction of a table; and having doubtless seen seven figure logarithmic tables (and perhaps heard of Briggs's imperfect fourteen figure table and Vlacq's ten figure table), he determined to devote his time to the independent calculation of a complete twelve figure table, mainly on the ground that it would be larger than any that had been calculated before.

During all Mr. Thomson's working lifetime Hutton's logarithms were the standard table, and it seems improbable that he should not have possessed a copy of so well-known a work; which, moreover, was well calculated to awaken interest in the subject on account of its full historical introduction. But all the early editions contained Sharp's table of logarithms of primes under 1,100, which Mr. Thomson does not appear to have seen, and of which he certainly made no use. At all events, whether as the result of want of knowledge, or of a strong desire to do everything with his own hand without being under obligation to any other computer, it seems pretty clear that he undertook and accomplished successfully by himself the whole labour of constructing a much larger table of logarithms of numbers than any that has been published, entirely from first principles, as if no logarithms had been calculated previously. There are, no doubt, cases on record, of persons who, from mere love of computing (*i.e.* delight in performing the calculations, rather than desire for the attainment of the results on account of their value) have performed similar pieces of work, and there are at present not a few unfinished MS. tables of logarithms in private hands that have been so undertaken; but there seems some reason to attribute to Mr. Thomson a higher object, and to suppose that having determined to construct the table, he carefully considered what its extent should be, so as to render it as valuable as possible relatively to the work bestowed on it. The extension of the table to 120,000 instead of 100,000 may, of course, have been

prompted merely by a desire to excel what had been done, but it may also have resulted from a knowledge of the inconveniences that attend the use of the existing tables, when the first figures begin with 10... or 11 ... and second-differences have to be taken into account.

It being thus nearly certain that the primes under 4,000 were computed afresh, it is highly probable that the others were so too, but the manner in which this was done is not very evident. Supposing the logarithms of all the composite numbers to have been found, written out, and (wherever possible) differenced a first and second time, it is clear that it would generally have been easy to have then obtained the logarithms of the primes by mere addition from the differences. I have therefore carefully examined the two books a_1 and a_2 that contain the differences, to see what internal evidence there is in favour of or against this course having been followed, but I have not been able to find anything that throws any light on the matter. In not a few cases there were corrections both in the numbers and differences, but these were always capable of explanation in both ways, viz. either on the supposition that the primes were copied from some other MS. book and differenced for a verification, or that they were obtained originally by means of the differences. In many places the logarithms of the primes and the corresponding differences have evidently been inserted at a later period than those of the composites in the vicinity; but this affords no clue to the mode of calculation of the former, as, however they were obtained, it is certain that the work was kept quite distinct from that relating to the composites. I rather incline to the opinion that the logarithms of the primes were not calculated from the second differences, but were obtained by some more independent method and differenced merely as a verification, the same as was the case with the composites. In both a_1 and a_2 the primes are marked with a pencil P,* and near the end of a_2 many of the composite numbers have one of their factors written before them. In the other MSS. P appended to a number *generally* denotes that it is a prime.

* The numbers so marked are not, however, all primes. I read the two chiliads 79,000–81,000 with the primes in Chernac, and found the following composites marked with a pencil P.:—

79,369 (=139·571)	79,789 (= 73·1093)	80,773 (=7·11·1049)
79,463 (=229·347)	79,921 (=229·349)	80,837 (=229·353)
79,667 (=7·19·599)	80,131 (=227·353)	

while 80,701, which is a prime, was not marked. It seems unquestionable that the pencil P was intended to denote a prime, and we must either suppose that for some reason the above composites were treated as primes in the calculation, or that Mr. Thomson believed them to be primes, or, what comes to the same thing, found it less trouble to treat them as primes than to determine whether they were so or not. In any case, the inference is, that Mr. Thomson had not in his possession an extended factor table or list of primes.

The factor table *e*, extending to 21,460,† may have been the result of original work, as although Maseres (1795), Vega (1797), Lambert (1798), and others, have given such tables up to 100,000 or 102,000, while Chernac's table (1811) extends to over a million, and Burckhardt's (1817) to three millions, Mr. Thomson may not have been acquainted with these works. With regard to the primes, besides the full lists contained in nearly all the works just mentioned, Barlow, in his *Mathematical Tables* (1814), gave a list of primes up to over 100,000, and in *Rees's Cyclopædia* (1819) there is a list up to 217,219, so that tables of primes were readily accessible in this country at the time Mr. Thomson was engaged upon his calculations.

The book A_4, containing the final portion of the completed results, is ruled for the logarithms throughout, and the numbers are written in as far as 123,187, which occupies the last line in the book; but the logarithms are only entered regularly (*i.e.* without omissions) as far as 120,005, above which there are very numerous blanks. It is evident, however, that Mr. Thomson had all along fixed 120,000 as the limit of the table, as the work is throughout performed on the supposition that higher multiples would not be required, and it is probable that he contemplated the addition of the next 3,000 logarithms, merely because there happened to be room for them in the book A_4. Beyond 120,000 we have, as it were, the table in process of formation, and no doubt a very attentive study might help to elucidate more fully the mode of procedure followed; but an ordinary examination shows nothing that could not be pretty nearly as well seen from the earlier portions of the table.

No one who examines the MSS. can fail to be struck by the great uniformity and neatness of the work. Though the calculation must have extended over several, and perhaps many years, and much of it must have been performed when Mr. Thomson was fatigued, there are no traces of any change of style or signs of haste; the figures are invariably made with the same clearness, and of the same uniform size. The work seems to have been done throughout in a slow, methodical way, and very accurately (viz. the alterations are very few). It is also everywhere condensed; for had the logarithms been copied out and so added, instead of the slips in *d* being used, the extent of the MSS. would have been tripled. This ought to be noticed particularly, as anyone merely looking at the MSS. might easily much underrate the amount of labour involved. To show the closeness of the writing, it is worthy of remark, that the four volumes *A* contain more figures than does Vlacq's large folio of 1628. One singular peculiarity in the work is, that Mr.

† It is a coincidence that Köhler's *Logarithmisch-trigonometrisches Handbuch* (Leipzig) contains a factor table up to 21,524, differing very little from Mr. Thomson's limit, viz., 21,460. The copy of Köhler that I have seen is of the second stereotyped edition (1848), and I do not know when the book was first published.

Thomson never writes a logarithm without its index; the columns are always headed "Number—Index—Logarithm." It seems strange that, for the sake of a fancy, he should have been willing to make so many thousands of useless figures, but this scrupulousness in a mere matter of form is, to a certain extent, characteristic of the work. Mr. Thomson had, no doubt, the logarithms continually present to his mind as exponents, and he was reluctant to commit to writing anything that from his point of view did not seem to be absolute truth. The attention so laboriously and perseveringly paid to a crotchet would not be inconsistent with the character of one who had resolutely determined to perform the whole of the work he had proposed to himself without assistance of any kind from others.

Mr. Cranston mentions that some of the MSS. are known to have been destroyed, and it seems curious that all the work that has escaped should have reference solely to the composites; but this may be explained by the consideration that the calculations relating to the primes, being less capable of methodical arrangement, may not have been so carefully written in books.

The non-logarithmic MSS. of Mr. Thomson's, to which allusion has been made (and which, including the few leaves containing the logarithms of powers of primes, I have numbered in red ink from 1 to 16), do not possess any value. Most of them relate to chemistry, and consist of extracts from printed books (chiefly Ure's *Dictionary of Chemistry*), on the composition of vegetables, a chemical notation, specific gravities of gases, velocities of winds, weights and measures of different countries, &c. &c., besides some formulæ, and a small table relating to the fall of bodies *in vacuo*. The longest is entitled "Murray's Experiments Illustrative of Chemical Science." The whole weight of these non-logarithmic MSS. is just two ounces, and they are only of interest in so far as they throw light on Mr. Thomson's early tastes. Most of them were probably copied at a time much anterior to that at which he commenced the calculation of his logarithms.

§ 2. *Accuracy of the MSS.; Errors found in Briggs's hundred and first Chiliad.*

Assuming, therefore, as can scarcely admit of doubt, that Mr. Thomson's twelve figure table to 120,000 is the result of a wholly original calculation, the next points to be considered are:—(1) how far it may be relied on for accuracy; and (2) to what uses it may be put.

To determine the accuracy of the MSS. I have read parts of them with the existing tables that give more than 12 figures; but before stating the results of these comparisons, it is desirable to explain briefly the contents of the books, &c., which give the most extensive tables of logarithms of numbers that have appeared, and the corrections thereto. These works, which therefore come in close relation with Mr. Thomson's MSS., are—

(1) Briggs's *Arithmetica Logarithmica*, folio, London, 1624.

Fourteen figure logarithms of numbers from 1 to 20,000, and from 90,000 to 100,000. A very few rare copies give an additional chiliad, and extend to 101,000.

(2) Vlacq's *Arithmetica Logarithmica*, folio, Gouda, 1628 (and London, 1631, with an English title). Ten figure logarithms of numbers from 1 to 100,000.

(3) Vega's *Thesaurus Logarithmorum Completus*, folio, Leipzig, 1794. Reprint of Vlacq, with 1,000 new logarithms added, viz. ten figure logarithms to 101,000.

(4) Borda and Delambre's *Tables Trigonométriques Décimales*, 4to, Paris, 1800. Contains eleven figure logarithms of numbers from 100,000 to 102,000.

(5) The *Tables du Cadastre*, calculated by the French Government at the end of the last century, contain fourteen figure logarithms from 1 to 200,000, the result of an original calculation. There are two copies of the MS., one at the Paris Observatory, the other at the Institute.

(6) The *Annales de l'Observatoire de Paris*, t. iv. (1858), contains the results of a reading, by M. Lefort, of the *Tables du Cadastre* in the Paris Observatory library, with Briggs, 1624, and Vlacq, 1628. List of errata so found in both these works are given.

One or two other MSS., of which no use has been made in this report, will be referred to further on.

It will be seen that 30,000 logarithms given by Briggs in 1624 (or 31,000 in the special copies), to fourteen places, were recalculated by Mr. Thomson to two places less. I therefore selected three groups of a thousand each (viz. 0–1,000, 19,000–20,000, and 100,000–101,000), and had them read from the MS., while I followed in Briggs.

Results of the Comparison of the first thousand Logarithms in Briggs and Thomson.

As Briggs extends to fourteen places, while Thomson has only twelve, the former gives always the next two figures, and affords the means of testing the accuracy of the last figure in the MS. Leaving out for the moment the cases in which the last two figures in Briggs were 50, there were found to be thirteen cases in which (on the assumption that Briggs was correct) the last figure ought to have been increased by a unit. Of these thirteen cases the next figures were 51 in four, and 52, 53, 54, 57,* 59, 60, 61,* 63,* 66 in the other nine, cases. The logarithms of the first 1,200 numbers are given in Callet, and therefrom it appears that in one of these cases, viz. in log 933, where Briggs gives the last two figures as 51, the figures after the twelfth really are 49994... ; so that Thomson is wrong twelve times, the remaining discrepancy being due to Briggs. The number of cases in which the last two

* On reference to Callet, it is seen that 57, 61, and 63 should be 58, 60, and 62.

figures in Briggs were 50 amounted to ten, in none of which was Thomson's last figure increased; but on referring to Callet for figures beyond the fourteenth it appeared that Thomson was only wrong three times. Thomson has five times written $\frac{1}{2}$ at the end of the logarithms, and in these cases the next figures were really 62, 61, 66, 50, 59. It is thus seen that the MS. up to 1,000 is very accurate, even in the last figure, and that the largest *avoidable* error amounts to .32 of a unit in this last place. It will be noticed that, as was found with the primes, the MS. is always in error by defect. The logarithms from 0 to 1,000, forming the beginning of A_1, are not written nearly so neatly as those in the rest of the MS., corrections being numerous and not always clear. Thus the last two figures of log 651, which are 68, cannot be distinguished from 08.

The comparison detected three errors in Briggs, of which the first (in log 80) is corrected in Lefort's list in the *Ann. de l'Obs. de Paris*, and the last (in log 540) by Briggs himself in his own errata list; but the remaining one, viz. in log 234 (at the top of the column), where the first figure of the logarithm should be a 3, instead of a 2, seems, very curiously, not to have been noticed. Briggs gives a hundred logarithms on each page in three columns, and the logarithms which appear at the foot of the first two columns (and which correspond to numbers ending in 34 and 67), are repeated at the head of the next columns. But in making a comparison between Briggs and another work, these logarithms that occur twice would not be read twice, so that in a case where one contained a misprint, which did not appear in the other, the error might easily escape detection, and this is what has no doubt happened with this misprint in log 234.

Results of the Comparison of the Logarithms of Numbers from 19,000 *to* 20,000 *in Briggs and Thomson.*

Briggs gives no logarithms of numbers between 20,000 and 90,000, so that 19,000–20,000 is the nearest chiliad to the middle of the table that could be selected. The comparison of this portion showed that (supposing Briggs to be always correct in his last two figures, and omitting the cases where they were 50), the number of last figure errors in Thomson was 47, which were thus distributed:—

Amount.	Number of errors, positive.	Number of errors, negative.	Total.
·50– ·60	7	12	19
·60– ·70	1	9	10
·70– ·80	5	1	6
·80– ·90	4	2	6
·90–1·00	0	1	1
1·00–1·40	2	3	5
	19	28	47

The ranges in the amount column are to be taken inclusive of the first limit, and exclusive of the second, thus: ·50—·60 includes any error whose first figure is a 5. An error is considered positive when the erroneous value exceeds the true value, viz.:

$$\text{error} = \text{erroneous value} - \text{true value}.$$

It is to be remarked that the negative are more numerous than the positive errors; that is to say, that the values in the MS. are generally too small. The largest errors (1·39 and 1·40) occur in logs 19,727 and 19,801. In nine cases in Briggs the last two figures are 50, and in but one of these the twelfth figure is increased in Thomson. It is not worth determining whether the increase should have been made or not, as for all practical purposes both are equally correct (see *Monthly Notices*, vol. xxxii. p. 261, and vol. xxxiii. p. 450, May 1872, and May 1873). A computer who wishes to be quite accurate, even with a perfect table, must so arrange his work that $x, y, z \ldots$ being the logarithms, and the decimal point referring to fractions of a unit in the last place,

$$f(x, y, z \ldots) = f(x + .5, y + .5, z + .5 \ldots)$$

to the number of figures he intends to retain, and he cannot make so delicate a distinction as is involved in discriminating between this and

$$f(x, y, z \ldots) = f(x + .55, y + .55, z + .55 \ldots)$$

Thus, both from the points of view of the calculator and the user of the table, discrepancies where the next figures are 50, 50 ± 1, or even 50 ± 5, cannot practically be regarded as errors. But as, throughout a good deal of his work, Mr. Thomson has only gone to twelve places, he does not in these cases deserve the credit for accuracy that belongs to those who reject additional figures, and therefore *only* make "errors" of this kind. In one instance (log 19,052) there is a 5 inserted at the end; the next two figures are really 56.

In this chiliad occurs the only real blunder (*i.e.* error, not in the last figure, and produced by the mode of calculation) that I have met with in the three chiliads (but two more in another part of the table will be pointed out in the next section). The logarithm of 19,620 is ·292 699 003 044, and not ·292 690 003 044, as given by Thomson. The book a_1 commences at 29,651, so that it cannot be decided whether this error was passed in the differences. The logarithm of 1962 is correctly given: it is possible that the error may have arisen in transcribing log 1962, where the tail of the 9 is a little separated from the body. The comparison brought to light ten errors in Briggs, but they were all found to be corrected in his errata list. I may mention that Briggs gives an erratum in a logarithm (log 19,677) in this chiliad that can only apply to

a part of the impression, as in two copies before me this logarithm is accurately printed, and needs no correction.

Results of the Comparison of the Logarithms of Numbers from 100,000 *to* 101,000 *in Briggs and Thomson.*

As mentioned above, in a few copies of Briggs there is an additional chiliad at the end containing the logarithms of the numbers from 100,000 to 101,000, and it seemed preferable to read this last chiliad with the MS. rather than any other, not only on account of its being the concluding portion of the fourteen figure table, which has never been carried beyond 101,000, or indeed extended since Briggs left it, but also because, even supposing that Mr. Thomson ever did possess or see a Briggs, it is very unlikely that he should have lighted upon one of the rare copies that contained this supplement. Supposing, as before, the last two figures of Briggs to be always correct, the number of last-figure errors (and there were no others) found in Thomson was 115, which were thus distributed :—

Amount.	Number of errors, positive.	Number of errors, negative.	Total.
·50– ·60	18	21	39
·60– ·70	19	13	32
·70– ·80	8	6	14
·80– ·90	9	6	15
·90–1·00	0	3	3
1·00–1·23	8	4	12
	62	53	115

So that in this chiliad the values in the MS. were generally too large. The largest error (1·23) occurs in log 100,461. In eight instances the last two figures in Briggs were 50, and in four of these the twelfth figure was increased in Thomson.

Briggs in his own errata-list only gave errors found in the portion of the table below 20,000; and Lefort's list, formed by comparison with the *Tables du Cadastre*, contains none above 100,000; in fact, there can be no doubt that neither Lefort nor Prony (who first had the comparison made) possessed a copy of Briggs containing the extension beyond 100,000. The following errata that were detected in the hundred and first chiliad of Briggs have therefore probably not been published previously :—

Number.	Error.	Correction.
100,008	37421	·47421
100,253	77377	73774
100,359	63262	63252
100,499	17203	17403
100,555	00239	00240
100,584	89027	89025
100,834	66956	69956

The last error is repeated, as 100,834 occurs both at the bottom of the first column and at the top of the second; but the latter (viz. the logarithm at the top of the second column) contains a fresh error, the last four figures being printed 2792 instead of 2782, as it is clear they should be from the differences. In all the seven cases noted above the differences were found to be correct. There is a very important printer's error affecting the whole of the last column but one in the chiliad, except the first and last logarithms, viz., extending from log 100,934 to log 100,966. This is caused by 100,934 being printed twice over in the number column, and 100,966 being omitted; thus, except the first and last, all the numbers in the column should be raised a line. By the comparison with Mr. Thomson's MS. the differences were of course not verified, neither were the thirteenth and fourteenth figures of the logarithms; the following errors in the differences were, however, met with while making the comparisons referred to hereafter:—

Number.	Error.	Correction.
100,119	6110	6119
100,482	43320	43220

Also, besides the erroneous column of numbers already noticed, errors were remarked in the following numbers:—

100,155	100,690	100,855	100,990
100,190	100,755	100,890	
100,655	100,790	100,954	

On the whole, then, the examination of the three chiliads disclosed only one blunder in the MS., and no last-figure error exceeding a unit by contraction (the largest being 1·40). Leaving out the cases where the next two figures in Briggs were 50, at the commencement of the table 98·8 per cent. of the logarithms were found to be correct; at 19,500, 95·3 per cent.; and at 100,500, 88·5 per cent.; so that the last figure throughout is generally to be relied upon, and appears never to be wrong by more than a unit. It is noticeable, also, that although the number of errors becomes greater as the numbers increase, their magnitude shows no tendency to do so, at all events above 20,000.

It is almost needless to remark that, leaving last-figure errors out of consideration, it would not be fair to say that Thomson was more accurate than Briggs, as most of the errors in the latter are due to the printer.

§ 3. *Errors found in Briggs, Borda, Vlacq, and Vega, by comparison with Mr. Thomson's MS.*

The characteristic features of the MS. are, that the logarithms are given to twelve places, and that they extend to 120,000; and not until a new ten figure table—with range 12,000 to

120,000—is in course of publication, can all the advantages which the MS. affords be brought into use. To the editor of such a table Mr. Thomson's work would be of much service, and enable him to guarantee the accuracy of the last figures, and also of the 20,000 additional logarithms. At present, however, the only purpose to which it can be put is the detection of errors in the existing tables, and I have accordingly so applied it in the manner which will be now described.

Considering first only the logarithms of numbers that exceed 100,000, the two tables in need of verification are Borda and Delambre, and Vega. As soon as the decimal division was declared by the French Government to be one of the fundamental articles of the new system of weights and measures, Borda applied himself to the calculation of new decimal tables, and in 1792 had completed his MS. Several causes delayed its publication, and at the time of his death a portion was still in the press. The printing was completed, and an introduction written, by Delambre, who edited the work in 1800 (an. ix.) under the title *Tables Trigonométriques Décimales.* Among the tables is one giving to eleven places the logarithms of the numbers from 100,000 to 102,000. There is no information to be found regarding it, but there can be no doubt that it was calculated by Borda himself. The first thousand logarithms (100,000-101,000) were given in the special copies of Briggs to fourteen places, so that over this portion the comparison was made between Briggs and Borda, and the corresponding values in Thomson were then referred to for confirmation of Briggs. (The comparison of this chiliad between Briggs and Thomson had been already made; see § 2.) The following were the discrepancies:—

No.	Borda.	Briggs.	Thomson.
100,119	8	7500	75
175	1	0474	05
231	1	1515	15
296	5	5524	55
331	8	8508	85
376	9	8482	84
378	6	5164	52
399	4	4522	45
489	3	3551	36
490	2	2517	25
534	0	0539	05
543	5	5534	55
592	0	0610	06
618	4	3499	35
649	0	0525	05
676	9	8453	85

No.	Borda.	Briggs.	Thomson.
100,679	5	4264	43
688	1	0339	03
697	10	09428	094
727	4	4500	45
747	7	7500	75
880	6	6504	65
939	7	7508	75
942	1	1507	15

There is also an important misprint in log 100,803, which appears in Borda as 00347245 . . . instead of 00347345 . . . When the next figures are 500, Borda is quite as likely to be right as not, and in any case he is guilty of no error. It will be observed that as far as possible Briggs is confirmed by Thomson, so that in most cases there can be no doubt of the true result. I calculated log 100,618 to fourteen places, and (assuming Briggs's value of log 7187 to be correct) found the last four figures to be 3498. The comparison of the chiliad from 101,000 to 102,000 showed the following discrepancies:—

Number.	Borda.	Thomson.	
101,079	8	86	(8446)
179	5	56	(5561)
194	5	56	(5575)
241	1	17	(1692)
767	10	094	(09381)

I calculated the logarithms of these five numbers to fourteen places, and have placed the last four figures in parentheses in the right hand column; they confirm Thomson's values in every instance except the first. In 115 cases in this chiliad the final figure in Thomson was 5, and in sixty-two of these the final figure was increased by contraction, and in fifty-three not increased, so that the accuracy of Borda's calculation is here also manifest.

The comparison of this chiliad detected two large errors in Thomson, viz. log 101,051 is given as ·00444 . . . instead of ·00454 . . . and log 101,068 as ·00451 . . . instead of ·00461 . . . These errors—no doubt due to a false addition—are not shown in a_2, as only the last six figures are there differenced.

In his *Thesaurus Logarithmorum Completus* (fol. 1794), Vega reprinted Vlacq's ten-figure table, and added a fresh chiliad, viz. from 100,000 to 101,000, which was calculated for him by Lieut. Dorfmund. This chiliad I read with Briggs, and then referred in all cases of discrepancy to Thomson for confirmation. The result showed that in no less than 280 instances was the last figure erroneous by a unit, and that in three instances it was in

error by two units. The logarithms of the 147 numbers contained in the following lists are too small by a unit in Vega, viz. the values given by Vega should be increased by a unit:—

100,021	100,148	100,363	100,539	100,737	100,862
022	149	365	563	739	863
024	153	368	564	743	865
025	165	369	566	744	867
035	167	371	568	774	868
036	211	373	569	778	883
038	213	382	585	779	884
045	214	383	589	801	886
047	236	405	591	802	888
058	237	406	592	804	889
059	262	408	593	805	895
061	264	457	594	806	896
074	266	461	595	807	914
075	267	462	596	808	916
077	268	463	597	809	917
078	269	464	599	811	918
088	271	465	603	812	919
089	272	466	654	841	921
091	287	467	655	842	922
104	288	491	657	848	944
105	296	492	733	849	945
107	297	496	734	851	947
118	301	535	735	853	956
119	361	536	736	854	957
121	362	538			

The logarithms of the 133 numbers contained in the following list are too large by a unit in Vega, viz. the values given by Vega should be diminished by a unit:—

100,003	100,141	100,279	100,485	100,679	100,788
004	142	307	486	691	817
007	143	328	487	692	824
009	144	330	494	693	834
041	157	333	502	694	835
051	159	334	542	695	836
052	183	335	544	712	837
065	184	336	545	714	874
066	185	337	546	726	875

100,095	100,186	100,338	100,554	100,727	100,904
096	187	339	555	728	905
097	188	356	556	754	906
098	222	357	557	756	907
111	223	358	558	764	908
112	224	377	559	765	932
124	225	412	575	766	933
129	226	413	576	767	934
132	227	414	577	768	979
133	228	415	578	769	981
134	252	472	635	785	993
136	253	474	637	786	994
137	277	484	677	787	995
138					

The logarithms of the three numbers,

100,364 100,366 100,367

as they appear in Vega, should be increased by two units in the last place.

In every one of the 283 discrepancies Briggs is confirmed by Mr. Thomson's MS., so that no doubt can exist about the above errata, but in seven instances the confirmation is doubtful, viz., for the numbers 100,007; 100,157; 100,357; 100,367; 100,545; 100,694; 100,824, in which the last four figures in Briggs are 4976; 4992; 4965; 5034; 4940; 4949; 4961 respectively; while in Thomson the last two are in each case 50. Most of the errata are such as Vega should have avoided, as in only eighty cases is the eleventh figure 4 or 5. It is curious that an editor who devoted so much labour to secure accuracy in the portion of the table that he reprinted from Vlacq should not have taken steps to ensure more correctness in the part for which he was himself responsible; or, at all events, should not have stated how far his last figure was trustworthy. There is one misprint in the chiliad, viz., the last two figures of log 100,456 should be 12 instead of 72.

But, after all, the tables of logarithms of numbers above 100,000 are, on account of their limited extent, and for other reasons, but little used; and the chief value of the MS. lies in its affording a verification of the ordinary ten-figure table to 100,000, and especially of the last figures. In 1858 M. Lefort read the whole of Vlacq with the *Tables du Cadastre*, and published in the *Ann. de l'Obs. de Paris* (t. iv.) a list of all the errors so detected in the former. This list is of the highest value to the user of ten-figure logarithms, and by its preparation and publication M. Lefort has anticipated a very great benefit, which otherwise the present MS. might have been made to confer upon

calculators. Had not M. Lefort made the comparison with the *Tables du Cadastre*, there can be no question that it would have been most desirable, and well worth the labour, to have read Mr. Thomson's MS. with Vlacq; but as it is, considering that such an examination could merely bring to light the few stray errors that may have escaped M. Lefort, it is very doubtful whether it is worth while to read nearly a million and a half of figures merely for this purpose. It is also to be noted that, although in making the comparison any clerk will do to read the numbers, it is essential that he who "hearkens" should be perfectly trustworthy, and have more than a perfunctory interest in the attainment of accuracy. Thus, although anyone editing a reprint of Vlacq would feel no doubt at all about the expediency of reading his proofs with Mr. Thomson's MS., it does not appear necessary to undertake a piece of work that would occupy so much time, at present, when there is no immediate prospect of a reprint.

But there is one important service that the MS. can render without any great expenditure of time being required, viz. the verification of the accuracy of M. Lefort's list. I accordingly had the whole of the logarithms indicated as erroneous by M. Lefort examined in Vlacq and the MS., and found that in every case (with one or two insignificant exceptions, to be mentioned further on) the latter confirmed the *Tables du Cadastre*. The errors in Vlacq are of two kinds, viz. large errors, chiefly due to misprints, and at once seen to be such when pointed out; and last-figure errors, due to the calculation not having been carried far enough to invariably secure complete accuracy to ten places. I have always very much regretted that M. Lefort in his list—no doubt to save space—did no more than merely indicate the error and correction in the latter case, so that the reader had no opportunity of inferring anything with regard to the amounts of the errors, or deciding between those that were important and those that were not. In the *Notices* for May 10, 1872, I remarked that "probably a good many of Lefort's errata were of this class," viz. due to the contraction in cases where the next figure was 5, and therefore of very little practical consequence; and anyone who uses ten-figure logarithms, and corrects his table by M. Lefort's list, must feel a desire for information as to the magnitude of the errors. I therefore carefully went through the whole list, selected every correction that had reference to the last figure, and placed side by side the last figure in Vlacq and the last three in the MS., inserting an additional column containing the errors and corrections as given by Lefort. In this way the following table was formed:—

Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.	Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.
10033	3	246	3	2	27586	8	872	8	9
11003	9	960	29	30	27861	2	268	2	3
11240	3	233	3	2	27921	7	633	7	6
15620	6	541	6	5	28486	9	952	699	700
17646	8	891	8	9	28680	9	996	69	70
17647	6	691	6	7	29112	5	578	5	6
17648	0	096	0	1	29163	8	850	8	9
17649	0	107	0	1	29226	9	956	799	800
20071	0	936	10	09	29446	7	751	7	8
20280	6	661	6	7	29639	8	750	8	7
20375	5	412	5	4	29703	3	231	3	2
20645	3	211	3	2	30499	0	950	6000	5999
20822	2	141	2	1	30502	8	740	8	7
20866	1	046	1	0	30728	1	157	1	2
21245	5	426	5	4	31001	2	108	2	1
21749	2	270	2	3	31627	5	579	5	6
21795	5	353	5	4	31653	6	756	6	8
21904	9	790	9	8	31735	6	650	6	7
22016	7	555	7	6	31817	9	955	79	80
22200	4	451	4	5	31919	8	749	8	7
22312	2	258	2	3	32111	5	566	5	6
22877	1	180	1	2	32633	9	958	09	10
22996	9	974	2999	3000	32672	5	407	5	4
23274	9	984	299	300	33370	6	655	6	7
23492	3	171	3	2	34037	6	654	6	7
23820	2	147	2	1	34162	4	350	4	3
24156	0	937	10	09	34358	4	350	4	3
25173	9	811	9	8	34664	1	026	1	0
25524	9	950	59	60	34702	4	451	4	5
25586	5	551	5	6	34734	9	950	7999	8000
25707	5	550	5	6	35053	8	850	8	9
26004	3	367	3	4	35298	7	771	7	8
26188	2	250	2	3	38051	9	741	9	7
26407	5	446	5	4	38277	1	153	1	2
26642	9	965	39	40	38321	7	623	7	6
26717	4	458	4	5	38578	6	754	6	8
27291	5	430	5	4	38783	3	231	3	2
27560	3	236	3	2	39227	4	454	4	5

Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.	Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.
39626	.	200	.	2	50601	7	650	7	6
39802	5	369	5	4	50828	3	250	3	2
39839	7	649	7	6	50937	1	050	1	0
40108	2	269	2	3	50996	5	450	5	4
40127	9	950	19	20	51037	3	250	3	2
40909	6	517	6	5	51096	2	150	2	1
40966	6	665	6	7	51175	4	335	4	3
41121	4	495	4	5	51388	5	450	5	4
41156	5	550	5	6	51389	7	650	7	6
41227	2	277	2	3	51606	1	050	1	0
41385	6	535	6	5	51607	6	550	6	5
42584	1	165	1	2	51820	7	650	7	6
44021	0	949	40	39	51915	4	350	4	3
44822	2	262	2	3	52064	2	257	2	3
45060	3	388	3	4	52533	8	749	8	7
45231	5	552	5	6	52565	8	750	8	7
45238	3	250	3	2	52587	8	749	8	7
45474	5	449	5	4	52620	8	750	8	7
45549	8	748	8	7	52792	3	354	3	4
45571	8	750	8	7	52823	7	650	7	6
45697	7	649	7	6	52986	2	150	2	1
45725	2	148	2	1	53050	3	237	3	2
45755	6	687	6	7	53647	8	750	8	7
45890	5	695	5	7	53868	5	449	5	4
46073	9	850	9	8	54026	3	250	3	2
47162	0	949	40	39	54145	1	050	1	0
47476	1	169	1	2	54419	0	951	70	69
48305	5	445	5	4	54586	8	866	8	9
48359	9	648	9	6	54708	3	250	3	2
48614	6	653	6	7	54825	4	350	4	3
48626	8	748	8	7	55010	0	950	50	49
48807	6	875	6	9	55115	8	750	8	7
48845	0	939	40	39	55313	9	850	9	8
48980	9	789	9	8	55996	8	864	8	9
49047	6	550	6	5	57089	8	750	8	7
49409	1	194	1	2	57202	7	649	7	6
50211	9	850	9	8	57486	6	550	6	5
50414	1	050	1	0	57751	8	750	8	7

Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.	Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.
58081	2	150	2	1	67399	0	949	30	29
58214	6	550	6	5	67686	9	608	9	6
58223	2	149	2	1	69311	7	773	7	8
58301	1	000	1	0	69457	3	250	3	2
58858	7	649	7	6	69477	5	450	5	4
59007	1	050	1	0	69988	2	150	2	1
59482	9	647	9	6	70019	0	949	40	39
59498	4	349	4	3	70040	3	411	3	4
60096	2	250	2	3	70043	1	004	1	0
60401	8	868	8	9	70066	7	750	7	8
60487	2	149	2	1	70599	6	528	6	5
60704	1	200	1	2	71140	9	850	9	8
61011	4	349	4	3	71306	9	834	9	8
61157	4	349	4	3	71518	5	918	5	9
61606	4	515	4	5	71569	0	064	0	1
62038	5	449	5	4	71653	3	351	3	4
62131	7	650	7	6	71764	6	526	6	5
62173	7	650	7	6	72103	9	849	9	8
62257	4	350	4	3	72675	5	442	5	4
62273	4	349	4	3	73046	0	950	90	89
62412	9	822	9	8	73059	4	482	4	5
62933	0	948	50	49	73286	2	251	2	3
63183	9	850	9	8	73303	0	950	90	89
63357	0	909	50	49	73404	6	545	6	5
63887	1	050	1	0	73501	9	812	9	8
64086	5	450	5	4	73570	1	042	1	0
64639	1	050	1	0	73571	2	148	2	1
64661	4	811	4	8	73655	9	849	9	8
64993	0	949	40	39	74527	6	550	6	5
65143	1	049	1	0	74723	8	749	8	7
65185	8	862	8	9	74733	5	450	5	4
65311	5	449	5	4	74932	5	449	5	4
65376	6	952	6	0	74941	0	949	40	39
65659	1	050	1	0	75149	9	850	9	8
65946	2	251	2	3	75386	2	150	2	1
66187	7	650	7	6	75395	6	546	6	5
66239	4	350	4	3	75560	3	250	3	2
66423	7	650	7	6	75562	4	453	4	5

Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.	Number.	Vlacq.	Thomson.	Lefort. Err.	Lefort. Corr.
75613	4	349	4	3	93155	1	173	1	2
75733	2	808	2	8	93202	7	878	7	9
75841	8	749	8	7	93498	0	052	0	1
75953	4	349	4	3	94468	7	922		9
77047	2	141	2	1	96453	7	151	7	2
77437	6	550	6	5	96981	0	943	80	79
77663	7	649	7	6	97361	2	907	2	9
77944	6	542	6	5	97674	5	552	5	6
78079	5	450	5	4	98336	5	576	5	6
78259	2	149	2	1	98337	9	988	49	50
79447	5	449	5	4	98338	3	355	3	4
79467	1	050	1	0	98339	6	677	6	7
79666	0	950	20	19	98340	9	954	39	40
80060	7	769	7	8	98341	1	186	1	2
80062	8	859	8	9	98342	3	373	3	4
80063	2	302	2	3	98345	6	666	6	7
80090	6	663	6	7	98346	6	674	6	7
80851	9	817	9	8	98348	5	554	5	6
81212	0	950	60	59	98350	2	255	2	3
81460	8	750	8	7	98352	7	777	7	8
82951	0	946	60	59	98353	4	470	4	5
82991	7	650	7	6	98356	2	281	2	3
83693	6	550	6	5	98357	7	794	7	8
83803	8	887	8	9	98358	2	264	2	3
85651	9	844	9	8	98359	6	688	6	7
85810	9	972	19	20	98360	0	067	0	1
86688	3	353	3	4	98362	6	690	6	7
86708	0	936	90	89	98365	2	289	2	3
86898	0	064	0	1	98366	3	399	3	4
87634	3	250	3	2	98367	4	464	4	5
89182		633	7	6	98772	8	736	8	7
89185		653	6	7	98936	7	780	7	8
90625	5	579	5	6	98966	4	339	4	3
91086	8	650	8	7	99926	2	115	2	1
91087	7	585	7	6					

A few words of explanation are required with regard to one or two numbers in this list. The last two figures of log 11,003 are in Vlacq 39, not 29, as given by Lefort, and the error may

have been merely a misprint of 9 for 0, or it may have been due to a misprint of 3 for 2, and an error of contraction. There is really no last-figure error in log 39,626, but the final 2 is imperfectly printed in some copies (see *Notices*, vol. xxxiii., pp. 453, 457, May 1873). The error in log 53,050 is assigned by Lefort to log 53,053, to which it could not belong. Mr. J. N. Lewis suggested (in the number of the *Notices* just cited) that the former logarithm was probably that which was meant; and as the correction is appropriate to it, and to no other in the vicinity—I had the last figures of the logarithms from 50,000 to 60,000 compared in Vlacq and Thomson—there can be no doubt that his explanation is the correct one.

Leaving out of consideration, for reasons already sufficiently explained, the cases where the last figures in the MS. are 50, there is only one instance in which Lefort's correction is not confirmed, viz. in log 54,419, where Thomson gives the eleventh and twelfth figures as 51. I accordingly calculated this logarithm *de novo* from the formula—

$$\log x = \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \frac{\text{modulus}}{2x^2-1} + \&c.$$

(54419 being a prime, $54418 = 2.7.13^2.23$, and $54420 = 10.2.3.907$), and found it to be ·73575 05569 49946 8 ... confirming the *Tables du Cadastre*. Of course, as we have seen before, Thomson's last figures are not to be trusted to a unit.*

* Since the account in the text was written Mr. Sang has sent me a copy of his paper *On Last-Place Errors in Vlacq's Table of Logarithms*, read before the Royal Society of Edinburgh, April 20, 1874, in which are given the results of a comparison between his own fifteen-figure MS. table (see §4) and Vlacq for the ten chiliads 20,000–30,000. This comparison was made in entire ignorance of what was done in 1858 by Lefort, for Mr. Sang writes: "It is indeed surprising that after so many years we are still relying on the unchecked calculations of Briggs and Vlacq; that among so many generations of scientific men there has not been zeal enough to effect a revision of the canon." And, further, in speaking of the *Tables du Cadastre*: "I have not learned that these computations have been used for the verification of those already printed, or that they have served for the production of any seven-place table; and thus, up to the present moment, we have no verification of Vlacq's great work;" so that the list of 42 errors detected in Vlacq by means of his MS., which Mr. Sang gives in his paper, is quite independent of any previous work. On comparing it with Lefort's list in the text, I find that there are only four errata indicated which do not appear in the latter; but in three of these cases (viz. in log 24,580, log 26,517 and log 26,728) the corrections have previously been published. The error in log 24,580 was pointed out in vol. xxxii., p. 258, (May 1872) of the *Notices*, and that in log 26,517, although not noticed by Lefort, was given by Vega in 1794 (see *Notices*, vol. xxxii., p. 288, June 1872); log 26,728 is in Lefort's list, but is omitted from that in the text, as not being a last-figure error. The other erratum seems to have escaped M. Lefort. It occurs in log 26,699, where the last figure should be a 3 instead of a 4, the succeeding figures being 4903... : this is confirmed by Thomson, who gives the last two figures as 49.

The corrections to log 26,188, log 29,112, and log 29,163 do not appear in Mr. Sang's list; and although the middle one is confirmed by Thomson, in the other two the last two figures are 50. I accordingly calculated those logarithms

It is rather curious to observe in the above list how little superior in accuracy Vlacq's last myriad is to the others which he calculated himself, although, as it was given by Briggs to 14 places, he might have avoided all errors in it. Considering that all the last-figure errors made in 90,000 results are contained in the list, it is manifest that Vlacq was very careful to be accurate in the last figures, and it seems strange that he should not have ensured absolute freedom from error, where it would have cost him nothing to have done so.

In order to see how far M. Lefort's list might be relied upon as complete, I had, as is incidentally mentioned above, the last figure in Vlacq examined with Thomson from 50,000 to 60,000. The result was the discovery, besides the corrections given by Lefort, of the following three discrepancies:—

No.	Vlacq.	Thomson.
51,141	8	874
52,743	7	648
59,205	5	449

On calculating these three logarithms to 14 places I found the figures after the tenth to be 7326; 5029; 5120 respectively, so that the first discrepancy (viz. in log 51,141) is due to a last-figure error in Vlacq that has hitherto escaped detection; but the other two are attributable to last-figure errors in the MS.; the logarithms as given by Vlacq being correct. It is proper here to warn any user of Mr. Thomson's MS. against his 3's, which, though very carefully made, are yet in cases very likely to be taken as 2's. Thus the antepenultimate figure in log 55,059 is either an excellent 3 or an excellent 2, according as it first strikes the eye. Other examples will be found in the antepenultimate figure of log 58,208 and the last figure but three of log 101,297. Several apparent discrepancies I found to be due to this cause.

§ 4. *Other Logarithmic MSS.—Publication of Mr. Thomson's MS.*

It still remains to notice some other unpublished logarithmic MSS. First in importance and celebrity stand the *Tables du*

to 14 places, and (taking the values of logs 6547, 7278, 9721 from Briggs) found the figures after the tenth to be 4995; 7818; 4997, confirming Mr. Sang in the first and third cases, and the *Tables du Cadastre* in the other, which probably inadvertently escaped Mr. Sang in the reading. M. Lefort states that the last two figures of the *Tables du Cadastre* are not to be relied upon; and this renders all his corrections where the last two figures in Thomson are 50 uncertain. The only cases where Thomson's last figure, as given in the list in the text, differs from the truth, between 20,000 and 30,000, are in 21,749, 21,795, 22,877, 24,156, 29,226, where the final three figures, according to Mr. Sang, should be 269; 354; 181; 936; 955. On the whole, however, it is satisfactory to obtain so good a confirmation of the general accuracy of M. Lefort's list in all essential points; and when comparing it with Mr. Thomson's MS. I little thought that further testimony as to its correctness was so soon to be added.

Cadastre, which have been referred to already. There is, however, also another MS., giving the logarithms of numbers above 100,000, which was calculated by the late Mr. G. H. Heppel, an actuary, who died in 1845. His tables contain the logarithms of numbers from 100,000 to 118,600 (to twelve places), and from 131,000 to 136,636, and 200,000 to 210,000 (to twelve or fifteen places). The first portion (100,000 to 118,600) *has been stereotyped*,* and the plates and MSS., Mr. Merrifield tells me, are now in the possession of Mr. C. Jellicoe, late President of the Institute of Actuaries. A seven-figure table of logarithms of numbers to 250,000 was calculated by Cartault or Cartaud, a Commis de la Marine, who died in 1784; in that year it passed into the hands of Lalande, as also did the table of logarithmic sines and tangents to every second which was calculated by Robert, Curé of Toul (see Lalande, *Bibl. Astron.*, pp. 667 and 688; Delambre, *Rapport Historique sur les Progrès des Sciences Mathématiques depuis* 1789 (1810), p. 64; and *Encyc. Méthod.* Art. *Tables Astronomiques*). Both passed into the hands of Delambre, and the former (viz. the logarithms of numbers) is now in the Graves Library, at University College, London. The sines and tangents were bought by Babbage, and are now in the possession of Lord Lindsay. There are in our own (the Astronomical Society's) library two volumes of MSS., the first of which relates to logarithms of numbers; they were calculated by the late Mr. T. W. Hill, F.R.A.S., and presented to the Society by his son, Mr. M. D. Hill, Q.C. The logarithms of numbers were intended, as appears from the title, to extend as far as 220,000 to 23 places of decimals (21 or 22 correct), and were commenced in the year 1806, the author's desire being to excel the *Tables du Cadastre*. The logarithmic work, although it fills a quarto work of moderate thickness, is far too incomplete to be of much value. To take an example:. On one page we have log 849 to 23 places, then to this log 2 is added eight times successively, giving the logarithms of 1698, 3396, . . . 217344; 851 and 853 are similarly treated. On another page further on log 3 is repeatedly added to the same three logarithms. The highest "fundamental" number is 1231, so that only the logarithms of multiples of numbers less than 1231 by powers of 2 and 3 are given; and any of these logarithms could easily be calculated *de novo* by a computer who required them. At the end of the volume is a factor table up to about 11,000. Had Mr. Thomson written down each time, as Mr. Hill has done, the logarithms he added (instead of using moveable slips), his MS. would have been of very great bulk. In the *Edinburgh Transactions*, vol. xxvi. (1872), Mr. Sang has described the manner in which he calculated a fifteen-figure table of the logarithms of numbers from 100,000 to 200,000, the first seven figures of which only he printed in his "New table of seven-place logarithms" (1871). He has been

* *Assurance Magazine*, vol. x. p. 82 (1863).

subsequently extending the table with the view of publishing a nine-figure table of logarithms up to 1,000,000. The reader of Mr. Sang's account will see that his mode of calculation in some respects resembles that which was probably pursued by Mr. Thomson.*

There can be no doubt that besides these MSS. there must be many others of more or less extent in private hands. It is not very uncommon for persons with a taste for mathematics to devote their leisure to the formation of logarithmic tables, though most likely there is no case in which so extensive a table as Mr. Thomson's has ever thus been produced. Either from the schemes proposed to themselves by their authors being too ambitious, or for other reasons, such tables are rarely completed, and would probably not be published, even if the necessary funds were forthcoming, as it is so much easier to calculate a table than to free from it errors when calculated. I have heard of two or three MSS. that have been constructed in this way. The celebrity attained by the French tables and the frequent expressions of regret in this country at their non-publication, coupled with the fact that the English Government once offered 5,000*l.* towards the printing, have no doubt several times, as was the case with Mr. Hill, incited computers to attempt to equal or surpass them, at least as far as the logarithms of numbers were concerned. But with regard to the *Tables du Cadastre*, it ought to be remembered that the logarithms of numbers only form a small portion of the whole MSS., and that the publication is, or rather was, urged on the ground, that the French Government, having undertaken the calculation, should complete its work by giving the results to the world. Were it a question of performing the whole work over again, it is very questionable whether many would be inclined to recommend a Government to undertake it at all.

The same causes that have prevented the publication of the *Tables du Cadastre* will probably prevent the publication of Mr. Thomson's MS. Such a table if printed would be very little used; and besides, anyone who desired to publish the whole would have to devote so much attention to verifications before he could be satisfied of its complete accuracy, that the work would not fall very far short of what would be required for re-computing all the portion not previously printed. No public body could well give funds towards the publication, as it is unquestionable that the money could be expended much more usefully, even in the matter of mathematical tables. It thus appears that until a ten-figure table to 120,000—a work really wanted, and one which, on account of its practical value and the public use that might be made of it, would be a proper devotion of national funds—no further use can be made of Mr. Thomson's MS., except for the detection of un-

* In the paper referred to in the note to § 3 Mr. Sang states that he has completed his fifteen-figure table up to 320,000, and that this portion, occupying 24 quarto volumes, was exhibited to the Royal Society of Edinburgh on April 20.

discovered errors in Vlacq. In this view I am supported by Mr. C. W. Merrifield, who writes to me: "Taking it for granted that Vega is unprocurable practically, what is chiefly needed is a republication of his work. I should be disposed to add 19 chiliads of common logarithms 101,000–120,000.... I think the question of very advanced logarithms, such as twelve-figure, lands us on a different step. For the present state of science I think an accessible reprint of the ten-figure table, circular as well as numerical, is required. The other is a desirable luxury, but only a luxury. Unless I can get it, I have no more right to a fourteen or twenty-figure table, because one exists, than I should have to *claim* the use of the 'Great Eastern,' because I wanted to try experiments on ships exceeding 600 feet in length. I attach importance to the 101st-120th chiliads. The second difference in ten figures only affects the last figure to the extent of 4 units at 120,000, and the break at 100,000 is a bad stopping-place, having regard to reciprocals. At the same time I do not think differences the proper mode of interpolation for any well-known and simple function like logarithms or circular functions. Adaptations of Taylor's theorem should always be used, except where proportional parts will do; but it is desirable to push the use of these without second differences as far as practicable, and for this the extension to 120,000 is very desirable."

Although computers would be very glad to have a twelve-figure table to refer to, it would be, as Mr. Merrifield observes, a luxury, and only a luxury, so that the MS. is not likely to be printed as it stands. I should recommend that the books containing the final results, viz. A_2, A_3, A_4, be bound together in one volume. A_1 being of smaller size, must be kept separate. The other volumes, containing the differences and the work, are of different sizes; and it would perhaps be desirable to let them remain as they are, so that anyone interested in the subject may be able to see them exactly as they were when in Mr. Thomson's possession.

It will have been seen that no printed table and only two MSS. (the *Tables du Cadastre* and Mr. Sang's) are more extensive than Mr. Thomson's table, which will henceforth be generally available for reference. Even had Mr. Thomson lived to complete his work, it is scarcely likely that it could have been put to any other use, besides that to which it has been applied in this account, viz. the verification of existing tables; and the thanks of all computers are due to Miss Thomson for having, by her valuable present, made so important a contribution towards the accuracy of logarithmic tables.

On the Combination of the different Results of various Series of Observations. By Dr. C. Powalky.

1. *Direct solution* for two series.

Let $[v_1]$ be the sum of all values v_1 of the first series; n_1 the number of its observations; $[d_1 d_1]$ the sum of the squares, $v_1 - \frac{[v_1]}{n_1}$;

Let $[v_2]$ be the sum of all values v_2 of the second series; n_2 the number of its observations; $[d_2 d_2]$ the sum of the squares, $v_2 - \frac{[v_2]}{n_2}$;

p_1, p_2 the weights.

The general equation is:

$$w = \frac{p_1[v_1]+p_2[v_2]}{p_1 n_1 + p_2 n_2} = \frac{[v_1]}{n_1} - \frac{\left(\frac{[v_1]}{n_1} - \frac{[v_2]}{n_2}\right) p_2 n_2}{p_1 n_1 + p_2 n_2} = \frac{[v_2]}{n_2} + \frac{\left(\frac{[v_1]}{n_1} - \frac{[v_2]}{n_2}\right) p_1 n_1}{p_1 n_1 + p_2 n_2}.$$

Putting now

$$\frac{[v_1]}{n_1} - \frac{[v_2]}{n_2} = \mathrm{D}; \; p_1 n_1 + p_2 n_2 = \mathrm{Z},$$

the squares of the errors are

$$[d_1 d_1] + n_1 \frac{\mathrm{D}^2 n_2^2 p_2^2}{\mathrm{Z}^2}; \; [d_2 d_2] + n_2 \frac{\mathrm{D}^2 n_1^2 p_1^2}{\mathrm{Z}^2};$$

and therefore

$$p_1 : p_2 = \frac{[d_2 d_2] + n_2 \frac{\mathrm{D}^2 n_1^2 p_1^2}{\mathrm{Z}^2}}{n_2 - 1} : \frac{[d_1 d_1] + n_1 \frac{\mathrm{D}^2 n_2^2 p_2^2}{\mathrm{Z}^2}}{n_1 - 1}$$

$$= (n_1 - 1)([d_2 d_2]\mathrm{Z}^2 + n_2 n_1^2 \mathrm{D}^2 p_1^2) : (n_2 - 1)([d_1 d_1]\mathrm{Z}^2 + n_1 n_2^2 \mathrm{D}^2 p_2^2),$$

or

$$(n_2 - 1) p_1 \{\mathrm{Z}^2 [d_1 d_1] + n_1 n_2^2 \mathrm{D}^2 p_2\} = (n_1 - 1) p_2 \{\mathrm{Z}^2 [d_2 d_2] + n_1^2 n_2 \mathrm{D}^2 p_1\}.$$

Taking $p_1 = q\, p_2$, we have

$$\mathrm{Z}^2 = p_2^2 (q n_1 + n_2)^2, \text{ and dividing by } p_2^3$$

$$(n_2 - 1) q \{(q n_1 + n_2)^2 [d_1 d_1] + n_1 n_2^2 \mathrm{D}^2\} = (n_1 - 1)\{(q n_1 + n_2)^2 [d_2 d_2] + n_1^2 n_2 \mathrm{D}^2\};$$

or

$$\alpha q^3 + \beta q^2 + \gamma q + \delta = 0.$$

If the problem is determinate, we find only one real value of q

from this equation, the other two being imaginary; but the problem is indeterminate if there are three real values.

2. The *indirect solution* is more practicable, and applicable to any number of series.

Take w as known, we have for the determination of $p_1, p_2, p_3 \ldots$

$$\frac{1}{p_1} = \left\{\left(w - \frac{[v_1]}{n_1}\right)^1 n_1 + [d_1 d_1]\right\} \div (n_1 - 1)$$

$$\frac{1}{p_2} = \left\{\left(w - \frac{[v_2]}{n_2}\right)^2 n_2 + [d_2 d_2]\right\} \div (n_2 - 1)$$

$$\frac{1}{p_3} = \left\{\left(w - \frac{[v_3]}{n_3}\right)^3 n^3 + [d_3 d_3]\right\} \div (n_3 - 1).$$

$$\vdots \qquad\qquad \vdots$$

Assuming for w any value for w', determine the weights for this, and determine—if w' is not right—another value w''; assuming now $w = w''$, determine a new value w''', and so on until $w^{(n+1)} = w^{(n)}$.

Example.

$\frac{[v_1]}{n_1} = +12{\cdot}8, n_1 = 9$; $\frac{[v_2]}{n_2} = +1{\cdot}5, n_2 = 4$; $\frac{[v_3]}{n_3} = +1{\cdot}6, n_3 = 5$; $\frac{[v_4]}{n_4} = +0{\cdot}5, n_4 = 7$.

$[d_1 d_1] = 40{\cdot}0$ $\quad$ $[d_2 d_2] = 1{\cdot}1$ $\quad$ $[d_3 d_3] = 12{\cdot}3$ $\quad$ $[d_4 d_4] = 12{\cdot}0$.

1. $w' = +6{\cdot}0$.

n	9	4	5	7	$[v]p$	np
$w - \frac{[v]}{n}$ . . .	−6·8	+4·5	+4·4	+5·5	+2·021	0·158
Squares . . .	46·24	20·25	19·37	30·26	+0·219	0·146
Multip. by n . .	416·2	81·0	96·9	211·8	+0·293	0·183
$+[dd]$ . . .	40·0	1·1	12·3	12·0	+0·094	0·188
Sum . . .	456·2	82·1	109·2	223·8	+2·627	0·675
$\div$ by $(n-1)$ . .	57·0	27·4	27·3	37·3		
$\log \frac{1}{p} =$ . . .	1·7559	1·4378	1·4362	1·5717		
$\log [v]$ . . .	2·0615	0·7782	0·9031	0·5441		
$\log p$	8·2441	8·5622	8·5638	8·4283		
$\log n$	0·9542	0·6021	0·6990	0·8451		

$w'' = +3{\cdot}9$

$$w'' = \frac{[v_1]p_1 + [v_2]p_2 + [v_3]p_3 + [v_4]p_4}{n_1 p_1 + n_2 p_2 + n_3 p_3 + n_4 p_4}$$

2. $w''=+3\cdot9$.

					$[v]p$	np
n	9	4	5	7		
$w-\frac{[v]}{n}$ . . .	−8·9	+2·4	+2·3	+3·9	+1·222	0·096
Squares . . .	79·22	5·76	5·29	11·56	+0·747	0·498
					+0·825	0·516
Multip. by n .	713·0	23·0	26·5	80·9	+0·226	0·451
$+[dd]$ . . .	40·0	1·1	12·3	12·0	+3·020	1·561
Sum . . .	753·0	24·1	38·8	92·9		
÷by $(n-1)$ . .	94·3	8·07	9·7	15·5	$w'''=+1\cdot93$	
$\log\frac{1}{p}=$ . . .	1·9743	0·9049	0·9868	1·1903		
$\log[v]$	2.0615	0·7782	0·9831	0·5441		
$\log p$	8·0257	9·0951	9·0132	8·8097		
$\log n$	0·9542	0·6021	0·6990	0·8451		

3. $w'''=+1\cdot93$.

					$[v]p$	np
$w-\frac{[v]}{n}=Q$.	−10·87	+0·43	+0·33	+1·43	+ 0·835	0·065
Q^2	118·1	0·185	0·109	2·045	+10·000	6·667
					+ 2·500	1·563
nQ^2	1062·9	0·74	0·54	14·3	+ 0·760	1·522
$+[dd]$ . . .	40·0	1·1	12·3	12·0	+14·095	9·817
Sum$=S$ . .	1102·9	1·84	12·84	26·3		
$\frac{S}{n-1}$	137·9	0·6	3·2	4·6	$w^{(4)}=+1\cdot44$	
$\log p=$ . . .	7·8604	0·2218	9·4949	9·3372		

4. $w^{(4)}=+1\cdot4$.

					$[v]p$	np
$w-\frac{[v]}{n}=Q$.	−11·4	−0·1	−0·2	+0·9	+ 0·76	0·06
Q^2	130·0	0·01	0·04	0·81	+16·22	10·81
					+ 2·58	1·61
nQ^2	1170·0	0·04	0·20	5·67	+ 1·17	2·33
$+[dd]$ . . .	40·0	1·1	12·3	12·0	+20·74	24·81
Sum$=S$ . .	1210·0	1·14	12·5	17·67		
$\frac{S}{n-1}$	151·0	0·38	3·18	2·94	$w^{(5)}=+1\cdot40$	
$\log p=$ . . .	7·8210	0·4318	9·5086	9·5229		

$p_1=0\cdot007$
$p_2=2\cdot703$
$p_3=0\cdot323$
$p_4=0\cdot333$

$w = +1{\cdot}40$ is here but one of the values; for, assuming first $w' = +11{\cdot}0$, we find $w'' = +11{\cdot}6$, $w''' = +11{\cdot}9$, and $w^{(4)} = w^{(5)} = +12{\cdot}0$ with the weights: $p_1 = 0{\cdot}170$, $p_2 = 0{\cdot}007$, $p_4 = 0{\cdot}063$. A third value of w will be found $= +9{\cdot}0$, and the weights $p_1 = 0{\cdot}047$, $p_2 = 0{\cdot}013$, $p_3 = 0{\cdot}014$, $p_4 = 0{\cdot}012$. The problem is indeterminate.

Excluding the first value of $\frac{[v]}{n}$ we obtain $w = +1{\cdot}38$, and no other value satisfying the conditions

$$w = \frac{p_2[v_2] + p_3[v_3] + p_4[v_4]}{n_2 p_2 + n_3 p_3 + n_4 p_4},$$

the weights are now:

$$p_2 = 3{\cdot}7,\ p_3 = 0{\cdot}32,\ p_4 = 0{\cdot}34.$$

The problem is determinate.

Washington, 1874, *May*.

Remark on the Influence of Errors of Observation on the Determination of the Orbit of a Planet from three Observations.

By Herr F. W. Berg.

(*Translation.*)

Let a be the semiaxis major of the orbit, e ($= \sin\phi$) the excentricity, c the mean anomaly of the epoch, ϖ the distance of perihelion from ascending node, Ω the longitude of ascending node, i the inclination to the ecliptic; and moreover let α and β be the longitude and latitude for the first observation, α' and β' for the second, and α'' and β'' for the third. Then, as is known, compare Gauss, *Theoria Motus*, Art. 76–76:

$$\begin{aligned} d\alpha &= \mathrm{A}\,da + \mathrm{E}\,de + \mathrm{C}\,dc + \mathrm{W}\,d\varpi + \mathrm{I}\,di + \mathrm{O}\,d\Omega, \\ d\beta &= \mathrm{a}\,da + \mathrm{e}\,de + \mathrm{c}\,dc + \mathrm{w}\,d\varpi + \mathrm{i}\,di + \mathrm{o}\,d\Omega, \end{aligned} \qquad (1)$$

and similarly for the other two observations.

The coefficients A, E, &c., depend on the time t and the elements a, e, &c., of the orbit, and we have thus in these equations the relation between the variations of α and β and those of the elements. From the equations (1) and the corresponding equations for the other two observations can be obtained new equations serving to express the variations of the elements in terms of the variations da, $d\beta$, da', $d\beta'$, da'', $d\beta''$. We obtain

$$\begin{aligned} \Delta da &= (a,a)\,da + (a',a)\,da' + (a'',a)\,da'' + (\beta,a)\,d\beta + (\beta',a)\,d\beta' + (\beta'',a)\,d\beta'', \\ \Delta dc &= (a,e)\,da + (a',e)\,da' + (a'',e)\,da'' + (\beta,e)\,d\beta + (\beta',e)\,d\beta' + (\beta'',e)\,d\beta'. \end{aligned} \qquad (2)$$

&c.

Considering now the quantities da, $d\beta$, &c., as errors of

observation, these it is clear will exercise the greatest influence on the values of the elements when the determinant Δ vanishes. To investigate when this happens: the value of the determinant is

$$\Delta = \begin{vmatrix} A\,a & A'\,a' & A''\,a'' \\ E\,e & E'\,e' & E''\,e'' \\ C\,c & C'\,c' & C''\,c'' \\ W\,w & W'\,w' & W''\,w'' \\ I\,i & I'\,i' & I''\,i'' \\ O\,o & O'\,o' & O''\,o'' \end{vmatrix}. \tag{3}$$

Let x, y, z be, for the first observation, the rectangular heliocentric co-ordinates of the planet; X, Y, Z those of the Earth; and ρ the distance of the Earth and planet; then

$$\begin{aligned} x &= \rho \cos\beta \cos\alpha + X, \\ y &= \rho \cos\beta \sin\alpha + Y, \\ z &= \rho \sin\beta + Z, \end{aligned} \tag{4}$$

and similarly for the second and third observations.

Then X, Y, Z being regarded as constant; we have (compare Gauss, *Theoria Motus*, Art. 76):

$$\begin{aligned} d\alpha &= o\,dz + \frac{\cos\alpha}{\rho\cos\beta}\,dy - \frac{\sin\alpha}{\rho\cos\beta}\,dx, \\ d\beta &= \frac{\cos\beta}{\rho}\,dz - \frac{\sin\alpha\sin\beta}{\rho}\,dy - \frac{\sin\beta\cos\alpha}{\rho}\,dx. \end{aligned} \tag{5}$$

If now v be the true anomaly of the planet, and r its radius vector (both for the first observation) the co-ordinates x, y, z can also be expressed as follows:

$$\begin{aligned} x &= r\{\cos(v+\varpi)\cos\Omega - \sin(v+\varpi)\sin\Omega\cos i\}, \\ y &= r\{\cos(v+\varpi)\sin\Omega + \cos(v+\varpi)\cos\Omega\cos i\}, \\ z &= r.\sin(v+\varpi)\sin i; \end{aligned} \tag{6}$$

and from these at once

$$\begin{aligned} \frac{dx}{d\Omega} &= -y, & \frac{dx}{di} &= z\sin\Omega, \\ \frac{dy}{d\Omega} &= x, & \frac{dy}{di} &= -z\cos\Omega, \\ \frac{dz}{d\Omega} &= o, & \frac{dz}{di} &= z\cos z; \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{dx}{d\varpi} &= -y\cos i - x\cos\Omega\sin i, \\ \frac{dy}{d\varpi} &= x\cos i - z\sin\Omega\sin i, \\ \frac{dz}{d\varpi} &= (x\cos\Omega + y\sin\Omega)\sin i; \end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{dx}{dc} &= \frac{a}{\cos\phi}\left\{\frac{p}{r^2}\frac{dx}{d\varpi} + e\sin v\frac{x}{r}\right\},\\
\frac{dx}{da} &= \frac{x}{a} - \frac{3mt}{a}\frac{dx}{dc},\\
\frac{dx}{de} &= -a\frac{x}{r}\cos v + \frac{p+r}{\cos^2\phi}\frac{\sin v}{r}\frac{dx}{d\varpi},
\end{aligned} \qquad (9)$$

where m is the mean daily motion of the planet, and t is the time; also $p = a\cos^2\phi$, $= \alpha(1-e^2)$. In the last three equations (9), x may be changed into y, or into z. In the equations (6), (7), (8), and (9), we may for x, y, z, write x', y', z', or x'', y'', z'', changing r, v, t into r', v', t', r'', v'', t'' accordingly.

From these partial variations of the co-ordinates in respect to the elements, the total variations may be deduced; we have

$$dx = \frac{dx}{da}da + \frac{dx}{de}de + \frac{dx}{dc}dc + \frac{dx}{d\varpi}d\varpi + \frac{dx}{di}di + \frac{dx}{d\Omega}d\Omega. \qquad (10)$$

&c.

Introducing this and the like expressions for y, z, x', y', z', x'', y'', z'', into the expressions (5), we obtain the variations $d\alpha$, $d\beta$, da', &c., expressed in the terms of the variations da, de, dc, &c., as the equations (1) require.

The formation of the values of the coefficients A, a, &c., is also easy. We have, for example:

$$\begin{aligned}
\mathrm{A} &= \frac{1}{\rho}\left\{\frac{\cos\alpha}{\cos\beta}\frac{dy}{da} - \frac{\sin\alpha}{\cos\beta}\frac{dx}{da}\right\},\\
\mathrm{A}' &= \frac{1}{\rho'}\left\{\frac{\cos\alpha'}{\cos\beta'}\frac{dy'}{da} - \frac{\sin\alpha'}{\cos\beta'}\frac{dx'}{da}\right\}.
\end{aligned} \qquad (11)$$

&c.

In order to make the determinants more manageable, the terms may be collected as follows; write

$$\left|\begin{matrix}\mathrm{A},\ \mathrm{a}\\ \mathrm{E},\ \mathrm{e}\end{matrix}\right| = \mathrm{P}, \quad \left|\begin{matrix}\mathrm{C},\ \mathrm{c}\\ \mathrm{W},\ \mathrm{w}\end{matrix}\right| = \mathrm{Q}, \quad \left|\begin{matrix}\mathrm{I},\ \mathrm{i}\\ \mathrm{O},\ \mathrm{o}\end{matrix}\right| = \mathrm{T}; \qquad (12)$$

and similarly for P′, P″, &c.; viz. P = Ae − Ea, &c.; then the determinant may be written:

$$\Delta = \left|\begin{matrix}\mathrm{P}, & \mathrm{P}', & \mathrm{P}''\\ \mathrm{Q}, & \mathrm{Q}', & \mathrm{Q}''\\ \mathrm{T}, & \mathrm{T}', & \mathrm{T}''\end{matrix}\right|, \qquad (13)$$

We hence obtain

$$\Delta = \mathrm{L}\left|\begin{matrix}\mathrm{M}\frac{\mathrm{R}}{\rho}\sin(\mathrm{A}-\Omega), & \mathrm{M}'\frac{\mathrm{R}'}{\rho'}\sin(\mathrm{A}'-\Omega), & \mathrm{M}''\frac{\mathrm{R}''}{\rho''}\sin(\mathrm{A}''-\Omega)\\ \mathrm{N}\frac{\mathrm{R}}{\rho}\sin(\mathrm{A}-\Omega), & \mathrm{N}'\frac{\mathrm{R}'}{\rho'}\sin(\mathrm{A}'-\Omega), & \mathrm{N}''\frac{\mathrm{R}''}{\rho''}\sin(\mathrm{A}''-\Omega)\\ \frac{\sin u\cos\psi}{\cos\beta}, & \frac{\sin u'\cos\psi'}{\cos\beta'}, & \frac{\sin u''\cos\psi''}{\cos\beta''};\end{matrix}\right| \qquad (14)$$

where

$$\mathrm{M} = \frac{p+r}{pr}\sin v - \frac{3amt}{2r^2\cos\psi}(e+\cos\epsilon)$$

$$\mathrm{M}' = \frac{p+r'}{pr'}\sin v' - \frac{3amt'}{2r'^2\cos\psi'}(e+\cos\epsilon')$$

$$\mathrm{M}'' = \frac{p+r''}{pr''}\sin v'' - \frac{3amt''}{2r''^2\cos\psi''}(e+\cos\epsilon'') \qquad (15)$$

$$\mathrm{N} = \frac{ae\sin v}{r\cos\psi},\quad \mathrm{N}' = \frac{ae\sin v'}{r'\cos\psi'},\quad \mathrm{N}'' = \frac{ae\sin v''}{r''\cos\psi''}$$

$$\mathrm{L} = -\left(\frac{r\,r'\,r''}{\rho\,\rho'\,\rho''}\right)^2 \sin^2 i\cos i;$$

and ϵ, ϵ', ϵ'' are the excentric anomalies, and u, u', u'' the arguments of latitude of the planet, R, R′, R″, the radius vectors of the Earth, A, A′, A″ its longitudes, and ψ, ψ', ψ'' the angles at the planet (in the triangle Sun, Planet, Earth), which, as is known, Gauss first introduced into the problem.

From the expression (14) it appears that the determinant Δ vanishes when

(1.) The planet in one of the three observations is at once in the ecliptic, and in opposition with the Sun, for then the corresponding A is $= \Omega$, and the corresponding $u = 0$.

(2.) $i = 0$, or $= \frac{\pi}{2}$.

(3.) $\psi = \psi' = \psi'' = \frac{\pi}{2}$: in such a case the passage from ρ to r is very uncertain.

(4.) $\mathrm{M}\,\mathrm{N}' - \mathrm{M}'\,\mathrm{N} = \mathrm{M}'\,\mathrm{N}'' - \mathrm{M}''\,\mathrm{N}' = \mathrm{M}'\,\mathrm{N} - \mathrm{M}\,\mathrm{N}'' = 0$: one of these three equations being always a consequence of the other two. From the conditions of this case relations may be obtained between the elements of the orbit, and the intervals θ, θ'', which may be introduced $t = t' - \theta$, $t'' = t' + \theta''$.

(5.) $e = 0$; viz., when the orbit is circular.

If the three positions of the planet and those of the Sun lie in one and the same great circle, the determinant is then very small; in such a case the difference between the longitudes of the Sun's positions and that of the node Ω is also very small, and the planet is then usually in opposition.

Observatory, Wilna, 1874, May 15/3.

Note on the Comparison of a Star in the Radcliffe Catalogue with others in the same Catalogue. By M. Antoine d'Abbadie.

Having undertaken last autumn to determine my colatitude by measuring, with a micrometer, zenith distances of stars at their passage through a fixed telescope placed vertically, I found on comparing No. 6100 of the Radcliffe Catalogue with four other stars of the same list and observed in the course of four different nights, that its N.P.D. was too small by a little less than 6″. Not presuming to set up my own authority against that of such an eminent observer as the late Radcliffe Astronomer, I requested Mons. Leverrier to get R. 6100 observed, stating merely that found its N.P.D. wrong. Three observations obtained at the Paris Observatory confirmed my result to a fraction of a second.

Although every observer is liable to mistakes, I am still diffident in this case, for the following reasons:—Johnson had four observations in N.P.D. If, as may be supposed, he reduced immediately one or two of them, and compared the result with that of Groombridge, he can scarcely have made, four times, the same mistake of 6″·3. Again, it is to be remarked that Groombridge obtained 46° 46′ 58″·8 in 1811·9, that 34·9 years later Johnson found 6″·3 *less*, and finally that in 1873·8, or 27 years later, M. Leverrier and myself agree in obtaining 5″·7 *more* than Johnson. This seems to indicate a period of slow change amounting to about 30 years. I would, therefore, recommend the observation of this star in N.P.D. to those who have suitable instruments. In Paris its R.A. has been found only 0ˢ·03 greater than that of Johnson.

On seeing the foregoing statement (communicated a the Meeting of June 1874) respecting the North Polar Distance of the star, No. 6100 of the Radcliffe Catalogue (identical with Groombridge 4083), Mr. Main immediately referred to the original computations, and found that an error of a very obvious character had been committed in taking the mean of two observations made in 1844, and two made in 1848, five seconds (the half of ten) having been omitted in the process. Such mistakes are not unfrequent, and arise from a desire to shorten the process of addition by not adding up the second column of figures. Still it is strange that the error was not detected in reading for the press, especially as the difference between this observed N.P.D. and that of Groombridge ought to have led to scrutiny. The printing was completed before Mr. Main took charge of the Observatory.

The erratum will stand as follows:—First Radcliffe Catalogue, page 351. No. 6100, Mean N.P.D. 1845·0, *for* 52″·5 *read* 57″·5.

Note on Sirius. By J. M. Wilson, Esq.

It may be worth while to point out that observations on *Sirius* as a double star are tending to confirm the result of spectroscopic examination of that star, as showing that its mass bears a less ratio to the Sun's mass than its brightness bears to the Sun's brightness; or, in other words, that it is intrinsically much brighter than the Sun, and therefore at a higher temperature.

The companion is not often within the reach of an $8\frac{1}{4}$-inch glass, but on one hazy evening last winter it occurred to me to turn the telescope to *Sirius*, and I saw one of the companions at once. Mr. Seabroke's measures agreed with my own, giving its position-angle 65°, and distance 11″. The measures were taken with a parallel-wire micrometer, without any contrivance for shutting off the light of the principal star, and with a power of about 400.

Mr. Gledhill has been good enough to furnish me with a few previously observed positions and distances of this companion:

	°		″		″
1863	88	and	7·6		
1865	76	,,	10·0	to	8
1866	71	,,	10·0	,,	9

Both the positions and distances, so far as they may be considered accurate, show that the companion is passing away from its periastre, and they are insufficient to determine with any approach to accuracy either the period or the mean distance; but taking 200 years and 11″ as a very rough guess, founded on these measures, and taking the parallax of *Sirius* as 0″·22, it follows that the distance of the companion of *Sirius* is 50 times the Earth's distance from the Sun, and, therefore, that the mass of *Sirius* $= \frac{50^3}{200^2} \times$ mass of Sun $= 3\frac{1}{8}$ mass of Sun.

But the amount of light given by *Sirius* has been estimated as more than 200 times that of the Sun, from which it would seem to follow that, even allowing for large errors in the elements above assumed, *Sirius* is much brighter intrinsically than the Sun, and, therefore, as the spectroscope tells us, at a higher temperature.

I venture to bring this Note before the Society in its very incomplete state, in order to attract attention to the companion of *Sirius*, and to show that it is within the reach of moderate instrumental power.

Temple Observatory, Rugby,
1874, *June* 9.

Observations, taken at Churt, of the Value of Colatitude given by η Ursæ Majoris on the Prime Vertical.

By R. C. Carrington, Esq., F.R.S.

Knowing that the *Nautical Almanac* position of *η Ursæ Majoris* was closely in accordance with numerous observations in the twelve-year and seven-year Greenwich Catalogues, I thought that it would give me a good determination of latitude, if observed in the prime vertical. I had had no previous trial of this method, but my altazimuth is perfectly suited for observation in this position, as in any other. On the 25th of April, 1874, I found that the reading of coincidence of the middle wire, with its image in a trough of mercury, illuminated by a Bohnenberger eye-piece, was

1. W. at Azim. 173° 10′	2. N. 263° 10′	3. E. 353° 10′	4. S. 83° 10′
rev. 58·3335	rev. 58·3295	rev. 58·3160	rev. 58·3110

from which I concluded the reading for collimation =0 to be $58^{rev.}\cdot323$, and so set it. The value of the wire being [1″·42941] gave the following corrections for level:

W.	N.	E.	S.
+0″·24	+0″·25	−0″·24	−0″·25

The value of the wires which will be required below were found to be:

1	2	3	4	5	6	7
[1·63203]	[1·53121]	[1·38947]	[−∞]	[1·38045]	[1·52808]	[1·63850]
−42ˢ·858	−33ˢ·979	−24ˢ·517	0ˢ·000	+24ˢ·013	+33ˢ·735	+43ˢ·501

and the side wires of *η Urs. Maj.* were computed by the formula given by Encke in the *Berlin Jahrbuch*, for 1843:

$$t'-t = \frac{\text{wire}}{\sin\delta\,.\,\cos\phi\,.\,\sin t} - \frac{1}{2}\cot t\,.\,15\sin 1''\left(\frac{\text{wire}}{\sin\delta\,.\,\cos\phi\,.\,\sin t}\right)^2 + \frac{1}{6}(1+3\cot^2 t)\,(15\,.\,\sin 1'')^2\left(\frac{\text{wire}}{\sin\delta\,.\,\cos\phi\,.\,\sin t}\right)^3;$$

where t for wire 1 in May 29, second observation is, for instance—

$$14^h\,43^m\,40^s - 13^h\,42^m\,31^s\cdot3 = 1^h\,1^m\,8^s\cdot7 = 15^\circ\,17'\,10''\cdot5,$$

w 1 is [1·63203], $l\sin t = 9\cdot42101$, $l\sin\delta = 9\cdot80858$, and $l\cos\phi = 9\cdot89132$.

The signs require attention; for instance, in the second observation of May 29

The first term of wires 1, 2, 3 is +, and for wires 5, 6, 7 −
„ second „ „ „ −, „ „ −
„ third „ „ „ +, „ „ −.

May 29. *η Ursæ Majoris*, observed at azimuth 263° 4′ 26″·35 :

Wires.	h	m	s	Correction. m	s	h	m	s
7	12	31	17·5	+4	53·60 =	12	36	11·10
6	12	32	21·6	+3	49·44 =			11·04
5	12	33	26·5	+2	44·60 =			11·10
4	12	36	10·5	0	0·00 =			10·50
3	12	39	6·3	−2	55·19 =			11·11
2	12	40	16·0	−4	4·96 =			11·04
1	12	41	23·5	−5	11·74 =			11·76
					Mean	12	36	11·09

Same star W. of meridian :

Wires.	h	m	s	Correction. m	s	h	m	s
1	14	43	40·0	+5	11·67 =	14	48	51·67
2	14	44	48·5	+4	4·85 =			53·35
3	14	45	57·5	+2	55·14 =			52·64
4	14	48	52·1	0	0·00 =			52·10
5	14	51	36·3	−2	44·59 =			51·71
6	14	52	42·5	−3	49·35 =			53·15
7	14	53	46·1	−4	53·54 =			52·56
					Mean	14	48	52·45

$\dot{\beta}$ Libræ, observed at azimuth 173° 4′ 23″·25 :

Wires.	h	m	s	Correction. s	h	m	s
1			24·8	+43·38 =			8·18
2			33·5	+34·39 =			7·89
3			42·4	+24·82 =			7·22
4			8·4	0·00 =			8·40
5			32·6	−24·31 =			8·29
6			42·2	−34·15 =			8·05
7	15	10	52·0	−44·03 =			7·97
				Mean	15	10	8·00

D. Ab.	Level.	Azim.			
s	s	s	h	m	s
+0·00	+0·01	−1·18 =	15	10	6·83

s
slow +8·69

Next I observed *η Ursæ Majoris* on June 1 at azimuth 83°4′31″·90:

Wires.	h	m	s	Correction. m	s	h	m	s
1	12	30	42·0	+4	46·75 =	12	35	28·75
2	12	31	41·5	+3	49·00 =			30·50
3	12	32	43·1	+2	46·42 =			29·52
4	12	35	30·0	0	0·00 =			30·00
5	12	38	19·8	−2	49·80 =			30·00
6	12	39	30·2	−4	0·65 =			29·55
7	12	40	43·0	−5	13·25 =			29·75
					Mean	12	35	29·72

Same star W. of meridian:

Wires.	h	m	s	Correction. m	s	h	m	s
7	14	44	13·0	+5	13·29 =	14	49	26·29
6	14	45	26·5	+4	0·65 =			27·15
5	14	46	37·5	+2	49·77 =			27·27
4	14	49	27·0	0	0·00 =			27·00
3	14	52	13·5	−2	46·42 =			27·08
2	14	53	15·5	−3	48·98 =			26·52
1	14	54	14·4	−4	46·77 =			27·63
					Mean	14	49	26·99

β Libræ observed at azimuth 173° 4′ 31″·8:

Wires.	h	m	s	Correction. s	h	m	s
1		...		...		...	
2			30·4	+34·39 =			4·79
3			39·5	+24·82 =			4·32
4			4·7	0·00 =			4·70
5			29·0	−24·31 =			4·69
6			38·8	−34·15 =			4·65
7	15	10	48·7	−44·03 =			4·67
				Mean	15	10	4·64

D. Ab.	Level.	Azim.			
s	s	s	h	m	s
+0·00	+0·01	−1·68 =	15	10	2·97

slow +12·56 s

Next I observed a complete transit of *Polaris* S.P. on June 4, at the same azimuth:

Wires.	h	m	s	Correction. m	s	h	m	s
7	12	42	6·0	+30	25·4 =	13	12	31·4
6	12	48	55·5	+23	35·6 =			31·1
5	12	55	42·0	+16	47·8 =			29·8
4	13	12	30·0	0	0·0 =			30·0
3	13	29	38·0	−17	8·8 =			29·2
2	13	36	16·0	−23	45·9 =			30·1
1	13	42	36·0	−29	58·4 =	13	12	37·6
			Mean			13	12	31·31

D. Ab.	Level.	Azim.
s.	s	s
+0·55	−0·52	−52·10
h m s		
= 13 11 39·24		slow +13·62

Arcturus, June 4:

Wires.	h	m	s	Correction. s	h	m	s
1			58·3	+45·56 =			43·86
2			8·0	+36·12 =			44·12
3			18·0	+26·06 =			44·06
4			44·0	0·00 =			44·00
5			9·6	−25·53 =			44·07
6			19·8	−35·86 =			43·94
7	14	10	30·1	−46·24 =			43·86
			Mean		14	9	43·99

D. Ab.	Level.	Azim.	
s	s	s	h m s
+0·00	+0·14	−1·06 =	14 9 43·07
			s
			slow +13·62

I found the value of azimuth by *Polaris* S.P. and *Arcturus* on June 4 to be $-28''\cdot51$, and accordingly assumed as the N. point all through

° ′ ″
173 4 3·3.

I now apply the data to η *Ursæ Majoris*, and find

N. & E.			N. & W.			S. & E.			S. & W.		
h	m	s	h	m	s	h	m	s	h	m	s
12	36	19·65	14	49	1·12	12	35	42·15	14	49	39·53

and remark that any error of azimuth is annihilated in taking them in pairs E. and W., and any error of level annihilated by taking the pairs N. and S.

Accordingly, May 29, we have as the half difference of η *Ursæ Majoris*,

h	m	s	°	′	″
1	6	20·735 =	16	35	11·01,

and the

N.P.D. of η *Ursæ Majoris* = 40 3 24·42;

from which we find as the value of ϕ the colatitude, by the common formula $\tan \phi = \cot \delta \,.\, \cos H$,

$$\phi = 38^\circ\ 52'\ 6{\cdot}65 \text{ by N.};$$

and from June 1 we have in the same way

$$1^h\ 6^m\ 58{\cdot}69^s = 16^\circ\ 44'\ 40{\cdot}04,$$

and the

$$\text{N.P.D. of } \eta \textit{ Ursæ Majoris} \text{ being} = 40^\circ\ 3'\ 23{\cdot}82,$$

$$\phi = 38^\circ\ 50'\ 19{\cdot}69 \text{ by S.}$$

Whence mean of N. and S.

$$\phi = 38^\circ\ 51'\ 13{\cdot}17,$$

a result in which I have little confidence, though I know of nothing but mere shifting of the instrument to affect it.

Note on the Polarisation of Coggia's Comet. By A. Cowper Ranyard, Esq.

On the 1st, 2nd, and 4th of July I examined the light of the comet with a double image prism, but could not with *certainty* detect a difference between the brightness of the two images. On the 6th I was for the first time able to satisfy myself that there were undoubted traces of polarisation: the component in excess being that in the plane passing through the Sun's estimated place below the horizon. The difference of brightness was best observed when the double image prism was turned so that the line joining the nuclei of the two images was at right angles to the axis of the tail, that is, at right angles to the line joining the Sun and Comet: 90° from this position the two images partly overlapped, and their relative intensity was not so easily compared; no traces of bands could be made out with a Savart, nor could I perceive any difference in tint between the two fields of a biquartz when placed in the principal focus of a 4-inch telescope, and examined with a Nicol's prism packed amongst the lenses of an erecting eye-piece.

I continued to examine the polarisation of the comet with a double image prism almost nightly until the 14th, and it appeared to me that the difference in the brightness of the two images continued to become more and more conspicuous. This was specially the case with respect to the relative intensity of the two images of the tail; but I should not like to speak with

certainty of the increase of polarisation of the envelopes in the immediate neighbourhood of the nucleus.

On the 13th I estimated that the brightness of the tail at a distance of three-fourths of a degree from the nucleus in one image was certainly more than double the brightness of the tail at a similar part of the other image. The relative brightness of the two images of the head of the comet might possibly be represented by the numbers 5 and 4; and the contrast between the brightness of the images of different portions of the tail seemed to increase with their distance from the nucleus.

The presence of a continuous spectrum in the tail of the comet precludes the idea of its being composed entirely of incandescent gas; and if it were merely a cloud of fine dust dispersing the Sun's rays, we should expect its light to be strongly polarized. I feel therefore driven to conclude either that the fine dust is incandescent, or that the individual particles, liquid or solid, which go to make up the continuous spectrum part of the tail are large compared with the wave-length of light.

Observations of Coggia's Comet (III. 1874). By E. J. Stone, Esq., M.A., F.R.S., Her Majesty's Astronomer, Cape of Good Hope.

The following observations of Coggia's Comet (III. 1874) were made by me on 1874, July 31. One of the comparison stars on this day was "299 Cape Catalogue 1860," or "Lacaille 3081." The mean places for 1874, January 1, are—

R.A.	N.P.D.
h m s	° ′ ″
7 52 39·06	119 59 50·69.

Cape Mean Times.	Diff. in R.A. Comet−Star.	Diff. in N.P.D. Comet−Star.	R.A. Comet.	N.P.D. Comet.
h m s	m s	′ ″	h m s	° ′ ″
17 10 31·7	+4 13·50	*5 3·98	7 56 51·95	120 4 46·14
17 22 57·5	+4 13·96	6 39·54	7 56 52·41	120 6 21·70
*17 33 52·6	+4 14·04	7 53·71	7 56 52·49	120 7 35·87

The differences in N.P.D. and R.A. have been corrected for refraction, but not for parallax.

The observations were made with the New Equatoreal by Simms.

The comet has been observed on July 29, 30, 31, and August 3, but the positions of the other comparison stars are not known at present.

The comet appeared of about the brightness of a third magnitude star.

* The times for this observation were entered from the clock by Mr. W. H. Finlay.

Spectroscopic and Meridional Observations of Coggia's Comet (III. 1874), made at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following spectroscopic observations of the comet were made by Mr. Christie:—

The comet was examined with a dispersive power of one compound prism (dispersion A to H $7\frac{1}{2}°$) and of two compound prisms, the spectroscope used being adapted to the Great Equatoreal. In the first case a plane reflector was used, to send the rays directly back after passing through the first half prism of the train (which consists of a half prism, four whole prisms, and another half prism silvered on the back, all compound); in the other case, the half prism silvered on the back was placed to receive the rays after passing through the first half prism. The spectrum of the comet was examined on July 3, 4, 7, 12, 13, and 14, and compared with the spectrum of dioxide of carbon, a vacuum tube containing it, being placed in the telescope two feet from the slit, the plan used by Dr. Huggins in his Stellar and Nebular observations. The spark was obtained from an induction coil, without a Leyden jar, and under these conditions the spectrum was very similar to that of the comet, consisting of three or more bright bands, on a continuous spectrum, which, with a slight change in the spark, became so bright as nearly to obliterate the bands. In the spectrum of the comet two bright bands were found by direct comparison on every occasion to be sensibly co-incident with the two brighter bands of dioxide of carbon; the position of the third band was not determined. All these bands were sharply terminated on the red side, and faded away gradually towards the blue, becoming at the same time shorter, in the direction transverse to the continuous spectrum. On July 7 the coma gave in addition to these bands a faint continuous spectrum, which afterwards seemed to become much brighter, so that the bands were quite obliterated on July 14; but this may have been caused by the twilight and haze as the comet approached the horizon. The spectrum of the nucleus was continuous, but it appeared to have traces of numerous bright bands, and three or four dark lines also were seen on several occasions, but owing to passing clouds they were lost before their positions could be determined. One appeared to lie between D and E, another on the blue side of *b*, and a third near F. On July 14 another strip of continuous spectrum was seen to the east of that of the nucleus, corresponding to the brighter side of the coma.

The nucleus on June 22 and July 6 was planetary, about 5″ in diameter; afterwards it seemed to contract and become nearly stellar.

The tail and coma were found to be partially polarised in a plane through the axis of the tail on July 3, 6, 7, and 13, a doubly

refracting prism being used; the polarisation of the tail appeared to be slightly greater than that of the coma.

The following naked-eye observations of the brightness of the nucleus and length of the tail were made:—

June 22. Nucleus brighter than * 5 mag. Tail about 3° long.
July 3. Nucleus considerably brighter than *o Ursæ Majoris*. Tail about 5°.
July 4. Tail about 7°.
July 5. Tail about 12° long, 2° broad at base, extending to *27 Ursæ Majoris*.
July 6 and 7. Tail about 15° long.
July 8. Nucleus = *β Ursæ Majoris* and < *α Ursæ Majoris*. Tail 16° or 17°.
July 9. Nucleus > *β Ursæ Majoris* but < *α Ursæ Majoris*.
July 12. Nucleus > *α Ursæ Majoris* or *α Persei*, perhaps = *α Aquilæ*. Tail extended fully up to *o Ursæ Majoris*.
July 13. Nucleus nearly = *Capella* at about same altitude > *α Ursæ Majoris* which was 25° higher, and also > *α Aquilæ*. Tail nearly 20° long.

The comet was also observed on the meridian with the Transit-circle on every available opportunity. The following are the resulting observed right ascensions and north polar distances, corrected for parallax:—

1874	Greenwich M.S.T. h m s	Observed R.A. h m s	Observed N.P.D. ° ′ ″
June 22	13 18 1·4	7 22 12·17	21 51 49·3
24	13 13 38·7	7 25 41·87	22 17 26·8
July 1	12 56 54·1	7 36 30·40	25 10 29·3
3	12 51 31·9	7 39 0·51	26 36 18·7
4	12 48 45·5	7 40 10·14	27 28 4·9
6	12 42 57·3	7 42 14·14	29 32 41·5
7	12 39 56·6	7 43 9·47	30 47 35·6
8	12 36 51·7	7 44 0·68	32 12 36·3
12	12 23 55·5	7 46 48·56	39 50 15·7

Royal Observatory, Greenwich,
1874, *Sept.* 8.

The Plate given with the present number belongs to Mr. Knobel's paper, "Observations of Jupiter, 1874," No. 8, pp. 403–409.

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